PART

5

Statistical Models for Forecasting and Planning

CHAPTER

16

Financial Calculations: Interest, Annuities and NPV

Outcomes

Financial information for decision making is important in every area of business. An understanding of basic financial calculations, such as *interest* and *annuities*, is essential for investments and loans. The *time value of money* is highlighted in this chapter by using interest and annuity calculations in financial tools, such as the net present value method, which is commonly used to evaluate investment projects.

After studying this chapter, you should be able to:

- explain the concept of simple and compound interest
- perform simple and compound interest calculations
- explain the terms *nominal* and *effective interest rates*
- understand the concept of annuities
- distinguish between different types of annuities
- perform various annuity calculations both manually and using *Excel*
- apply the net present value method to evaluate investment projects.

16.1 Introduction to Simple and Compound Interest

Interest is the price paid by a borrower for the use of a lump sum of money provided by a lender for a given period of time. It is, alternatively, the bonus received by an investor when a lump sum is deposited for a given period of time.

Interest on any lump sum borrowed or invested is always quoted at a given *rate per cent per annum*. Interest calculations are therefore exercises in percentage calculations. Since lump sums can be invested or borrowed for any period of time, interest calculations must also take the element of *time* into account.

There are two methods of working out the interest amount on any lump sum investment or loan. They are *simple interest* and *compound interest*. This chapter examines both of these methods of interest calculations and demonstrates their application to lump sum investments and periodic investment (annuities) situations.

The following terms are used in all interest calculations:

- The principal or **present value**, P_v. This is the *initial lump sum* invested or borrowed.
- The **term**, n. This is the *duration* of the investment or loan and can be quoted in days, weeks, months or years.
- The **rate of interest**, i. This rate is quoted as a *per cent per annum value*, but used as a decimal in calculations (e.g. 9% p.a. = 0.09).
- The amount or **future value**, F_v. This is the total amount of money available at the *end* of the investment or loan period. It is the sum of the present value and the accrued interest amount.

Interest calculation formulae consist of these four components, namely P_v, F_v, i and n. In order to find the value of any one of them, values must be available for the remaining three elements. When applying interest formulae to a problem, the element to be found must always be identified first.

Another important issue in interest calculations is that the quoted interest rate period must *coincide* with the time interval over which interest accrues over the duration of an investment or loan. This issue will be highlighted in the worked examples.

16.2 Simple Interest

If interest is calculated on the *original lump sum only for each period* over the duration of an investment or a loan, it is termed **simple interest**.

For a lump sum of money, P_v invested (loaned) for n periods at a rate of interest of i% per period, the future value due (owing), F_v at the end of the investment or loan period based on the simple interest principal is defined as:

$$F_v = P_v(1 + in) \qquad \textbf{16.1}$$

In each period, interest at the rate of i per cent on the original principal (present value), P_v, accrues. Its accumulation over n periods determines the simple interest amount that has accrued on the investment or the debt.

Example 16.1 Fixed Deposit Investment (Simple Interest)

Mrs Hendricks invested R6 000 at 7% p.a. for three years in a fixed deposit account with the Cape City bank. If *simple interest* is paid at the end of three years on this deposit:

(a) how much money altogether will be paid out to Mrs Hendricks on maturity (i.e. at the end of three years?

(b) how much interest was earned over the three years?

Solution

(a) Required to find: the *future value*, F_v, of the fixed deposit investment.

Given $P_v = R6\ 000$ (principal invested)
$i = 0.07$ (rate of interest per annum)
$n = 3$ (the investment period, in years)

It should be noted that the interest rate period and the term coincide. They are both expressed in *years*.

Then
$$F_v = 6\ 000[1 + (0.07)(3)] \quad \text{(using Formula 16.1)}$$
$$= 6\ 000[1 + 0.21]$$
$$= 6\ 000[1.21]$$
$$= R7\ 260$$

Thus Mrs Hendricks will receive R7 260 at the end of three years.

(b) The difference between the future value, F_v (R7 260) and the present value, P_v (R6 000) of R1 260 is the amount of simple interest earned on the R6 000 invested.

Example 16.2 Fixed Deposit Investment (Principal Amount)

How much money should an investor deposit in a fixed deposit account paying 6% p.a. *simple interest* if she would like to receive R10 000 in four years' time?

Solution

Required to find: *the present value*, P_v, of the deposit.

Given $F_v = R10\ 000$ (the amount to be received at the end of four years)
$i = 0.06$ (rate of interest per annum)
$n = 4$ (the investment term, in years)

Again, note that the interest rate period and the term coincide, namely that they are both expressed in *years*.

Then
$$10\ 000 = P_v[1 + (0.06)(4)] \quad \text{(using Formula 16.1)}$$
$$= P_v[1.24]$$

So
$$P_v = \frac{10\ 000}{1.24}$$
$$= R8\ 064.52$$

Management Interpretation

The investor must deposit R8 064.52 in the fixed deposit account now in order to receive R10 000 in four years' time, based on simple interest of 6% p.a.

Again, the amount of the simple interest received over this four-year investment period is R1 935.48 (i.e. R10 000 – R8 064.52).

Example 16.3 Student Loan (Simple Interest Rate)

A student borrows R9 000 for three years at a *simple interest* rate to finance her management studies. If she must repay R11 295 at the end of the three-year loan period, what rate of simple interest per annum was she being charged by the bank?

Solution

Required to find: the *rate of simple interest*, *i*.

Given $F_v = R11\,295$ (the loan amount to be repaid after three years)
$n = 3$ (the term, in years)
$P_v = 9\,000$ (the loan amount)

Then
$$11\,295 = 9\,000\,[1 + i(3)] \quad \text{(using Formula 16.1)}$$
$$\frac{11\,295}{9\,000} = 1 + 3i$$
$$1 + 3i = 1.255$$
$$3i = 0.255$$
$$i = 0.085 \quad \text{(Convert to \% by multiplying by 100.)}$$

Management Interpretation

The *simple rate of interest* charged on the student loan was 8.5 % p.a. Since the *term* of the loan was quoted in units of *years* (i.e. three years), the period of the interest rate found must relate to the same time period. In this instance, the simple interest would be quoted as the rate *per annum*.

Example 16.4 Fixed Deposit Investment (Term of Investment)

An investor deposits R15 609 in a fixed deposit account that pays 6.25% p.a. *simple interest*. For how long must the amount be deposited if the investor wishes to withdraw R20 000 at the end of the investment period?

Solution

Required to find: the *term* (duration) of the investment, *n*.

Given $P_v = 15\,609$ (the principal amount deposited)
$F_v = 20\,000$ (the maturity value at the end of the term)
$i = 0.0625$ (the interest rate *per annum*)

Then
$$20\,000 = 15\,609(1 + 0.0625n) \quad \text{(using Formula 16.1)}$$
$$\frac{20\,000}{15\,609} = 1 + 0.0625n$$
$$1 + 0.0625n = 1.2813$$
$$0.0625n = 0.2813$$
$$n = 4.5 \text{ years}$$

Management Interpretation

The principal of R15 609 must be invested for 4.5 years at 6.25% p.a. *simple interest* to grow to R20 000 by the end of this investment period.

In all the above examples, the interval over which simple interest accrued corresponded to the unit of measure of the term. Both are quoted in *years*. Always ensure that the interest rate period coincides with the unit of measure of the term. The following two examples illustrate this requirement.

Example 16.5 Hire Purchase (Simple Interest Rate)

What is the *annual rate* of *simple interest* on the purchase of a stove costing R2 500, which is to be paid for in three months' time when R75 interest will be charged?

Solution

Required to find: the *rate of simple interest per annum*, *i*.

Given F_v = R2 500 + R75 = R2 575 (the future value owing in three months' time)
P_v = R2 500 (the initial purchase value)
$n = 0.25$ (three months, expressed in *yearly* terms)

Note: Since the required interest rate must be quoted in per cent *per annum*, the term must correspond and also be quoted in units of *years*.

Then

$$2\,575 = 2\,500[1 + i(0.25)] \quad \text{(using Formula 16.1)}$$

$$\frac{2\,575}{2\,500} = 1 + 0.25i$$

$$1 + 0.25i = 1.03$$

$$0.25i = 0.03$$

$$i = 0.12 \quad \text{(Convert to \% by multiplying by 100.)}$$

Management Interpretation

The simple rate of interest charged on the stove hire purchase agreement was 12% p.a.

Example 16.6 Short-term Business Loan (Simple Interest Rate)

An industrial printing company borrowed R75 000 from an investment bank to buy a new high-tech printer. The bank will charge *simple interest* on the bridging finance, which must be repaid in 18 months' time. If the printing company repaid R90 750 at the end of the loan period, what rate of simple interest *per annum* was being charged on the loan?

Solution

Required to find: the *rate of simple interest per annum*, *i*.

Given F_v = R90 750 (the future value owing in 18 months' time)
P_v = R75 000 (the purchase value of the high-tech printer)
$n = 1.5$ (18 months, expressed in *yearly* terms)

Note: Since the required interest rate must be quoted in per cent *per annum*, the term must correspond and also be quoted in units of *years*.

Then $90\,750 = 75\,000[1 + i(1.5)]$ (use Formula 16.1)

$$\frac{90\,750}{75\,000} = 1 + 1.5i$$

$$1 + 1.5i = 1.21$$

$$1.5i = 0.21$$

$$i = 0.14$$ (Convert to % by multiplying by 100.)

Management Interpretation

The industrial printing company was charged a simple rate of interest of 14% p.a. to finance the high-tech printer and repay the debt (price plus interest) in 18 months' time.

16.3 Compound Interest

Compound interest is the practice of calculating interest *periodically* and *adding* it to the existing principal *before* each subsequent interest calculation is made.

The interest is said to be *capitalised*. This means that it is made part of the present value on which interest is calculated each time. It is the practice of 'earning interest on interest'.

Development of the Compound Interest Formula

The period between two consecutive points in time at which interest is compounded is called the *conversion period*. At the end of each conversion period, interest is calculated on the accumulation of the original principal and all previous interest amounts. This is illustrated in Table 16.1. It shows that the future value at the end of each conversion period is the present value for the next period's interest calculation.

Table 16.1 The development of the compound interest formula

Period	Present value at the beginning of each period	Revised present value at the end of each period (after interest)		
1	P_v	$F_{v1} = F_v(1 + i)$	or	$P_v(1 + i)^1$
2	F_{v1}	$F_{v2} = F_{v1}(1 + i)$	or	$P_v(1 + i)^2$
3	F_{v2}	$F_{v3} = F_{v2}(1 + i)$	or	$P_v(1 + i)^3$
4	F_{v3}	$F_{v4} = F_{v3}(1 + i)$	or	$P_v(1 + i)^4$
5	F_{v4}	$F_{v5} = F_{v4}(1 + i)$	or	$P_v(1 + i)^5$
etc.	etc.	etc.		
n	$F_v(n - 1)$	$F_{vn} = F_{v(n-1)}(1 + i)$	or	**$P_v(1 + i)^n$**

Future Value

In general, the compound interest formula is used to derive the future value of an investment or loan after n periods at i% per annum, compounded *annually*. The formula is:

$$F_v = P_v (1 + i)^n \qquad \textbf{16.2}$$

Each symbol has the same definition as for simple interest.

If compounding occurs *more frequently* than once a year (i.e. monthly, quarterly), then Formula 16.2 is modified to show the quicker growth in interest earned or paid, as follows.

- The annual interest rate, i, is *divided* by the number of compounding periods in a year, k. If compounding occurs half-yearly, then $k = 2$; if quarterly, $k = 4$; if monthly, then $k = 12$.
- The term, n, which is expressed in years, is *multiplied* by the number of compounding periods in a year, k, to reflect the number of compounding periods over the duration of the term. If $k = 4$ (quarterly compounding), then $n \times 4$ is the total number of compounding periods over n years.

The modified formula is:

$$F_v = P_v \left(1 + \frac{i}{k}\right)^{nk} \qquad \textbf{16.3}$$

These two formulae can now be used to find any one of the following elements of a loan or investment for which interest is capitalised at regular intervals:

- future value, F_v
- present value, P_v
- interest rate, i
- term, n.

Example 16.7 Fixed Deposit Investment (Compound Interest)

Mrs Hendricks invested R6 000 in a fixed deposit account with the Cape City Bank for three years at a rate of interest of 7% p.a.

(a) How much will Mrs Hendricks receive upon maturity if interest is *compounded annually*?

(b) How much will Mrs Hendricks receive upon maturity if interest is *compounded quarterly*?

Solution

Required to find: the *future value*, F_v, of the fixed deposit investment.

(a)	Given	P_v = R6 000	(principal invested)
		$i = 0.07$	(rate of interest per annum)
		$n = 3$ years	(the investment period, in years)

Since interest is *compounded annually*, both the annual interest rate and the term must be expressed in annual units (i.e. 7% p.a. and three years).

Then $F_v = 6\,000(1 + 0.07)^3$ (using Formula 16.2)
$= 6\,000(1.07)^3$
$= 6\,000(1.225043)$
$= R7\,350.26$

Management Interpretation

Mrs Hendricks will receive R7 350.26 at the end of three years, based on interest compounded annually.

The difference between the future value, F_v (R7 350.26) and the present value, P_v (R6 000) of R1 350.26 is the amount of compound interest earned on the R6 000 invested.

(b) Given $P_v = R6\,000$ (principal invested)
$i = 0.0175$ (rate of interest per quarter)
$n = 12$ quarters (the investment period, in quarters)

Since interest is *compounded quarterly*, both the annual interest rate and the term must be expressed in *quarterly* units. The annual interest rate must be divided by 4 ($\frac{0.07}{4}$ = 0.0175 per quarter), while the term must be multiplied by 4 (3 years × 4 = 12 quarters). Thus an interest rate of 1.75% per quarter over 12 quarters applies.

Then $F_v = 6\,000(1 + 0.0175)^{12}$ (use Formula 16.3)
$= 6\,000(1.0175)^{12}$
$= 6\,000(1.231439)$
$= R7\,388.64$

Management Interpretation

Mrs Hendricks will receive R7 388.64 at the end of three years, based on interest compounded quarterly.

The difference between the future value, F_v (R7 388.64) and the present value, P_v (R6 000) of R1 388.64 is the amount of compound interest earned on the R6 000 invested.

Note the following:

- Compound interest always pays *more* than simple interest for the *same* capital amount, term and interest rate (the maturity value in Example 16.1 above was R7 260 when based on simple interest, while the maturity value when based on compound interest was R7 350.26 (compounded annually) and R7 388.64 (compounded quarterly)).
- The *amount* of compound interest earned/paid is always *larger* when the compounding period is more *frequent* (in Example 16.7(1), the amount of interest earned when compounded *annually* was R1 350.26, while in Example 16.7(2), the amount of interest earned when compounded *quarterly* over the same term of three years was R1 388.64. This is R38.38 more as a result of compounding quarterly instead of annually.

Present Value

When the initial amount (present value) of a loan or an investment for which interest is compounded is unknown, the compound interest formula (both formulae 16.2 and 16.3) can be re-arranged to find the present value as follows:

$$P_v = \frac{F_v}{(1+i)^n} \qquad \textbf{16.4}$$

Example 16.8 Fixed Deposit Investment (Monthly Compound Interest)

A father would like to give his daughter a cash gift of R20 000 on her 21st birthday in exactly four years' time. How much must he deposit today in a fixed deposit account that pays 9% p.a. *compounded monthly* to reach his target?

Solution

Required to find: the *present value*, P_v, of the deposit.

Given F_v = R20 000 (the maturity value required)
$i = 0.0075$ (rate of interest per month)
$n = 48$ months (the investment period, in months)

Since interest is *compounded monthly*, both the annual interest rate and the term must be expressed in *monthly* units. The annual interest rate must be divided by 12 ($\frac{0.09}{12} = 0.0075$ per month), while the term must be multiplied by 12 (4 years × 12 = 48 months). Thus an interest rate of 0.75% per month over 48 months applies.

Then $20\,000 = P_v(1 + 0.0075)^{48}$ (using Formula 16.4)

$$20\,000 = P_v(1.431405)$$

$$P_v = \frac{20\,000}{1.431405}$$

$$= R13\,972.38$$

Management Interpretation

The father must deposit R13 972.38 today into a fixed deposit account to receive R20 000 in four years' time, at 9% p.a. *compounded monthly*.

Interest Rate

When the *interest rate* of a loan or an investment where interest is compounded is unknown, the compound interest formula can be re-arranged to find this *interest rate* as follows:

$$i = \sqrt[n]{\frac{F_v}{P_v}} - 1 \qquad \textbf{16.5}$$

Example 16.9 Interest Payable on Loan (Half-yearly Compound Interest)

What is the annual rate of interest being charged by Good Hope Bank on a loan of R25 000 if R33 063 is to be repaid in two-and-half years' time in full settlement, if interest is compounded half-yearly?

Solution

Required to find: the *per annum interest rate*, *i*.

Given P_v = R20 000 (the original loan amount)
F_v = R33 063 (the amount to be repaid)
n = 5 half-yearly periods (2.5 years expressed in half-yearly periods)

Note: When a value for *i* is found, this will refer to the interest rate *per half year*.

Then
$$33\,063 = 25\,000(1 + i)^5$$
$$(1 + i)^5 = \frac{33\,063}{25\,000} = 1.32252$$
$$1 + i = \sqrt[5]{1.32252} = 1.0575$$
$$i = 1.0575 - 1$$
$$= 0.0575$$ (Convert to % by multiplying by 100.)

Management Interpretation

The half-yearly interest rate is 5.75%. This translates into an *annual* interest rate of 11.5% (i.e. 11.5% p.a.).

Term

To find the term of an investment or a loan, the compound interest formula can be re-arranged as follows:

$$n = \frac{\log\left(\frac{F_v}{P_v}\right)}{\log(1 + i)} \qquad 16.6$$

Note: The re-arranged formula requires the calculation of the *logarithm* of a number. The logarithm of a number to the base 10 can be found using the *Excel* function **=LOG(number)**.

Example 16.10 Investment Duration (Quarterly Compound Interest)

If an amount of R86 400 is invested at 8% p.a. compounded *quarterly*, how long will it take to reach R206 500.60?

Solution

Required to find: the *term*, *n*, of the investment.

Given P_v = R86 400 (the original investment amount)
F_v = R206 500.60 (the maturity value)
$i = 0.02$ (quarterly interest rate i.e. $\frac{0.08}{4} = 0.02$)

Note: When a value for *n* is found, this will refer to the number of *quarterly periods*.

$$\text{Then}\quad 206\,500.60 = 86\,400(1 + 0.02)^n$$
$$\frac{206\,500.60}{86\,400} = (1.02)^n$$
$$(1.02)^n = 2.390056$$

The term, n, can be separated from the number 1.02 by taking the natural logarithm of both sides of the formula, using either the *Excel* **LOG** function key or the log operation on a calculator.

$$n \log 1.02 = \log 2.390056$$
$$n(0.0086) = 0.378408$$
$$n = \frac{0.378408}{0.0086}$$
$$= 44 \text{ quarters} \qquad \text{(equivalent to 11 years)}$$

Management Interpretation

It would take 44 quarters (i.e. 11 years) for an investment of R86 400 to grow to R206 500.60, invested at 8% p.a. compounded quarterly.

16.4 Nominal and Effective Rates of Interest

All deposit and lending institutions quote an annual rate of interest on deposits or amounts loaned. This is called the *nominal* rate of interest. If interest on a deposit or a loan is compounded, then the *real* (or *effective*) rate of interest per annum will be different from the quoted, nominal rate of interest, due to the compounding effect.

Effective Rate of Interest

The following formula is used to determine the effective rate of interest on any investment or loan when interest is compounded in periods of less than a year.

$$r = \left(1 + \frac{i}{m}\right)^m - 1 \qquad \textbf{16.7}$$

Where: r = the effective interest rate per annum
m = the number of conversion periods per annum
i = the nominal interest rate per annum.

Example 16.11 Savings Account Investment (Monthly Compound Interest)

In an advertisement, ABSA Bank announced that a savings account for R100 000 or more would pay a nominal rate p.a. of 6.5%, capitalised monthly. What is the *effective* rate of interest p.a. earned on any deposit in this account?

Solution

Given $i = 0.065$ (the nominal interest rate p.a.)
$m = 12$ months (the number of conversion periods p.a.)

Then $r = \left(1 + \frac{0.065}{12}\right)^{12} - 1$ (using Formula 16.7)

$= (1.005417)^{12} - 1$

$= 1.066972 - 1$

$= 0.066972$ (6.6972%)

Management Interpretation

This ABSA Bank savings account pays an effective 6.6972% p.a. interest on deposits of R100 000 or more. This means that any amount of R100 000 or more invested for one year would have grown by 6.6972% and not 6.5%.

ABSA stipulated in its advertisement's terms and conditions that '*effective* rates apply for amounts invested for a period of one year, assuming the interest at the nominal rate is constant and capitalised monthly'.

16.5 Introduction to Annuities

When a constant amount of money is paid (or received) at *regular intervals* over a period of time, it is called an **annuity**.

When a lump sum of money is borrowed and then repaid over time (such as a property mortgage), or lump sums are invested and then withdrawn in fixed amounts at regular intervals (such as a retirement annuity), interest is constantly being charged or earned. Annuity calculations therefore involve an interest factor compounded at regular intervals. These series of equal payments could be made yearly, half-yearly, quarterly, monthly, weekly or even daily.

These are some common examples of annuities:

- regular monthly contributions into a pension fund (or provident fund)
- regular monthly contributions to a retirement annuity
- monthly insurance premiums on an endowment (or life) policy
- motor vehicle lease (or repayment) agreements
- monthly mortgage bond repayments
- hire-purchase agreements to settle outstanding debt
- unemployment insurance contributions
- short-term personal insurance policies (accident, car insurance, all risks, homeowner's insurance, hospital insurance plans) which can be paid monthly, half-yearly or annually.

Terminology in Annuities

The following terminology is used in all annuity calculations:

- The **future value** of an annuity, F_v. This is the amount of money that has accumulated by the maturity (or expiry) date of an annuity (i.e. when payments/contributions cease). It is equivalent to the sum of all regular payments, plus accumulated interest.
- The **present value** of an annuity, P_v. This is an initial lump sum of money that is either deposited or borrowed, and which will result in a series of equal payments at regular intervals for a period of time into the future.

- The term, n, is called the **duration** of an annuity.
- The **payment period** is the time interval between successive *regular payments* of *equal amounts*. The payment period must always coincide with the period over which interest is compounded.
- The **regular payments**, R. This is the amount of money that must be *deposited or paid at each payment period* over the duration of the annuity.
- The **rate of interest**, i, which must coincide with the conversion (or compounding) period.

Annuity calculations require finding one of the following:
- the future value of an annuity, F_v
- the present value of an annuity, P_v
- the regular payments, R.

The diagrams in Figure 16.1 illustrate the different concepts of annuities.

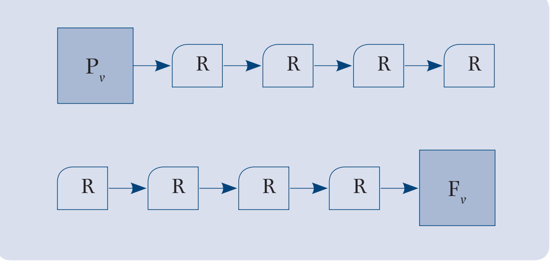

Figure 16.1 Illustration of the concepts of annuities

Classification of annuities

Annuities differ according to *when* the series of *regular payments begin*. In this regard, there are two broad groupings of annuities: *ordinary annuities* and *deferred annuities*.

Ordinary Annuities

For **ordinary annuities**, the first of the series of regular payments begin in the *first period* of the term of the annuity.

There are two kinds of ordinary annuities: ordinary annuity *certain* and ordinary annuity *due*.

An **ordinary annuity certain** is an annuity for which the series of regular payments take place at the *end* of each payment period for a fixed number of periods.

An **ordinary annuity due** is an annuity for which the series of regular payments take place at the *beginning* of each payment period for a fixed number of periods.

This difference in payment practice affects the amount of interest earned or charged on the ordinary annuity.

Deferred Annuities

Where the first of the series of regular payments only *begins* at some *future period*, and not immediately, during the term of the annuity, the annuity is known as a **deferred annuity**.

Illustration of a Deferred Annuity

A trust fund of R150 000 is instructed to pay the beneficiary, who is currently 12 years old, regular annual instalments over five years, beginning only when she turns 21. Since the beneficiary turns 21 years old in nine years' time, the annuity is said to be deferred for eight years (not nine years). The first payment will occur at the end of the ninth year, and four subsequent payments will take place. The beneficiary will receive the final payment from the trust when she is 25 years old.

16.6 Ordinary Annuity Certain

The regular payments under an ordinary annuity certain occur at the *end* of each payment period. This section derives the future value, F_v; the present value, P_v; and the regular payment, R, of an ordinary annuity certain. However, only the rationale of calculating future values of an ordinary annuity certain will be given to promote an understanding of the concept of an annuity and its utilisation of compound interest knowledge.

Example 16.12 Future Value of an Ordinary Annuity Certain

At the *end* of each year an investor makes six equal deposits of R500 into an investment account with the Cape City Bank, which pays 12% p.a. compounded annually.

Management Question

How much will the investor receive after six years?

Solution

To determine how much the investor will receive at the end of the sixth year, consider the graphic display of compound interest payments on each regular deposit of R500 over six years, as shown in Figure 16.2.

- The *first* deposit of R500 takes place only at the end of year 1. This R500 will earn interest for five years, at the rate of 12% p.a. compounded annually.
- The *second* deposit of R500 takes place at the end of year 2 only, and will earn interest for four years, at the rate of 12% p.a. compounded annually.
- The *third* deposit of R500 takes place at the end of year 3 only, and will earn interest for three years at the rate of 12% p.a. compounded annually.
- Similarly for the *fourth* and *fifth* deposits.

- Finally, the *sixth* and final deposit of R500 takes place at the end of year 6. It will earn no interest, as the full value of the investment (i.e. its *future value*) is due to be paid out on the same day that the final deposit is made.

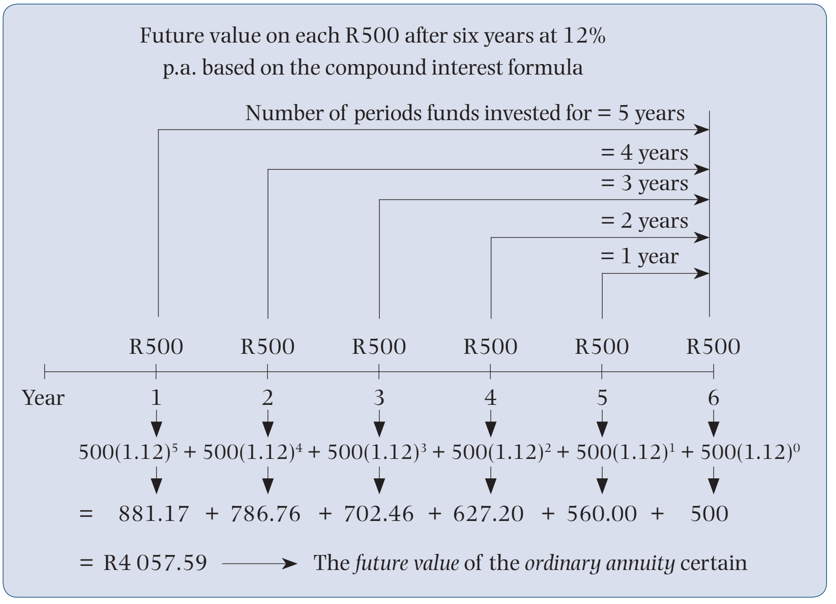

Figure 16.2 Concept of the future value of an ordinary annuity certain

As noted in the explanation, each regular payment (or deposit instalment) earns interest at a compounded rate for *that portion of the term for which it is invested*. As seen in the illustration, the *future value* of the *annuity* is the *sum* of all these regular payments *plus* the compound interest amount that each instalment has earned over the duration of the annuity.

Future Value of an Ordinary Annuity Certain

The *future value* of an *ordinary annuity certain* can be expressed in a mathematical formula as follows:

$$F_v = R\frac{(1+i)^n - 1}{i} \qquad \textbf{16.8}$$

Using the *illustration data*, the future value calculations based on Formula 16.8 are:

Given $i = 0.12$ (compounded rate p.a.)
$n = 6$ years
$R = 500$ (regular payment amounts)

Then $F_v = 500\frac{(1.12)^6 - 1}{0.12}$

$= 500(8.11519)$
$= R4\,057.59$

Management Interpretation

The investor will receive R4 057.59 at the end of six years, after investing R500 per year.

Example 16.13 Education Savings Plan (Monthly Compound Interest) (1)

A father decides to invest R650 at the *end* of each month for five years at 9% p.a. compounded monthly to pay for his son's tertiary education in five years' time. What amount of money will be available at the end of five years?

Solution

Since each payment takes place at the *end* of every month, this is an ordinary annuity certain.

Required to find: the *future value*, F_v, of an ordinary annuity certain.

Given $R = 650$ (regular monthly instalment)
$i = 0.0075$ (interest compounded monthly, i.e. $\frac{0.09}{12} = 0.0075$)
$n = 60$ months (number of regular payment periods = 5 years × 12 = 60 months)

Then
$$F_v = 650\,\frac{(1.0075)^{60} - 1}{0.0075} \quad \text{(using Formula 16.8)}$$
$$= 650\,\frac{0.565681}{0.0075}$$
$$= 650(75.42414)$$
$$= R49\,025.69$$

Management Interpretation

The father will have R49 025.69 available to pay for his son's tertiary education in five years' time.

Example 16.14 Car Purchase Savings Plan (Monthly Compound Interest)

Sam Gumede plans to buy a new car in two years' time, which he estimates will cost him R140 000. He also estimates that the trade-in value on his current car will be R36 500 in two years' time. How much must Sam save monthly in a special savings account with his bank that will pay 6.5% p.a. compounded monthly to buy the new car in two years' time?

Solution

Sam will require R103 500 (i.e. R140 000 – R36 500 trade-in) in two years' time for the purchase of a new car.

Required to find: the *regular payment*, R.

Given $F_v = 103\,500$ (future value required)
$i = 0.005417$ (monthly interest rate, i.e. $\frac{0.065}{12} = 0.005417$)
$n = 24$ months (number of regular payment periods = 2 years × 12 = 24 months)

Then
$$103\,500 = R\,\frac{(1.005417)^{24} - 1}{0.005417} \quad \text{(using Formula 16.8)}$$
$$= R\,\frac{0.138429}{0.005417}$$
$$= R(25.55611)$$
$$R = \frac{103\,500}{25.55611}$$
$$= R4\,049.91$$

Management Interpretation

Sam Gumede would have to save R4 049.91 at the *end* of every month for two years in order to meet the cost of the new car that he hopes to buy at that time.

Present Value of an Ordinary Annuity Certain

If the *present value* (initial lump sum) of an *ordinary annuity* certain is required to generate a regular flow of payments over a fixed period of time, then the following annuity formula is used:

$$P_v = R\frac{1-(1+i)^{-n}}{i} \qquad \textbf{16.9}$$

It must always first be established from the description of an annuity problem whether *present values* or *future values* are being referred to, as this determines which formula will be selected to solve the problem.

Example 16.15 Retirement Plan (Monthly Compound Interest)

Mr Peterson, who has just retired, would like to receive R5 000 at the *end of every month* for 10 years. If money can be invested at 9% p.a. compounded monthly, how much money should he deposit today?

Solution

Required to find: the *present value* of an ordinary annuity certain.

Given $R = 5\,000$ (regular monthly income)
$i = 0.0075$ (interest compounded monthly, i.e. $\frac{0.09}{12} = 0.0075$ per month)
$n = 120$ months (number of regular payment periods = 10 years × 12 = 120 months)

Then $P_v = 5\,000\frac{1-(1.0075)^{-120}}{0.0075}$ (using Formula 16.9)

$= 5\,000\frac{1-0.407937}{0.0075}$

$= 5\,000(78.94169)$

$= \text{R}394\,708.50$

Management Interpretation

Mr Peterson will have to deposit a lump sum of R394 708.50 today to ensure a retirement income of R5 000 per month for the next 10 years, payable at the end of each month.

Example 16.16 House Bond Repayment Scheme (Monthly Compound Interest)

Mr Ncube puts down a deposit of R50 000 on a new house costing R540 500. He has raised a mortgage bond for the balance, which he is required to pay off in equal monthly instalments over 20 years. Interest is charged at 10.5% p.a. compounded monthly. What regular monthly instalments will he have to pay at the *end* of each month?

Solution

From the description of the problem, it can be seen that the *present value* formula must be used since the question provides information on the *initial amount outstanding* (i.e. *present value*) and not on a future value.

Required to find: the *regular monthly payments*, R, on the mortgage bond.

Given $P_v = 490\ 500$ (outstanding balance after deposit, i.e. R540 500 – R50 000 = R490 500)

$i = 0.00875$ (interest compounded monthly, i.e. $\frac{0.105}{12} = 0.00875$ per month)

$n = 240$ months (number of regular payment periods = 20 years × 12 = 240 months)

Then $490\ 500 = R\frac{1-(1.00875)^{-240}}{0.00875}$ (using Formula 16.9)

$= R\frac{1-0.12358}{0.00875}$

$= R(100.1623)$

$R = \frac{490\ 500}{100.1623}$

$= R4\ 897.05$

Management Interpretation

Mr Ncube will have to repay R4 897.05 at the *end* of each month for 20 years to pay off the mortgage bond on his new house.

16.7 Ordinary Annuity Due

An *ordinary annuity due* is an annuity where the series of regular payments take place at the *beginning* of each payment period for a fixed number of periods. In such cases an *extra period's interest is earned or paid.*

Formulae for finding the *future value*, F_v, and *present value*, P_v, of an ordinary annuity due are similar to those for the ordinary annuity certain cases, but modified to take into account the extra period's interest that is earned or is owing.

Also, prior to performing ordinary annuity due calculations, it must be established whether the problem involves *future values* or *present values*, as this determines the choice of the appropriate formula.

Future Value of an Ordinary Annuity Due

The *future value*, F_v, for an *ordinary annuity due* is given as:

$$F_v = R\frac{[(1+i)^n - 1](1+i)}{i} \qquad 16.10$$

The additional term in the future value formula is $(1 + i)$, which accounts for the extra period's interest earned or owing.

Example 16.17 Education Savings Plan (Monthly Compound Interest) (2)

A father decides to invest R650 at the *beginning* of each month for five years at 9% p.a. compounded monthly to pay for his son's tertiary education in five years' time. What amount of money will be available at the end of five years?

Solution

Since each payment takes place at the *beginning* of every month, this is an *ordinary annuity due*.

Required to find: the *future value*, F_v, of an ordinary annuity due.

Given	$R = 650$	(regular monthly instalment)
	$i = 0.0075$	(interest compounded monthly, i.e. $\frac{0.09}{12} = 0.0075$)
	$n = 60$ months	(number of regular payment periods = 5 years × 12 = 60 months)

Using the future value Formula 16.10 above for an ordinary annuity due:

$$F_v = 650 \frac{[(1.0075)^{60} - 1](1 + 0.0075)}{0.0075}$$
$$= 650 \frac{(0.565681)(1.0075)}{0.0075}$$
$$= 650(75.98982)$$
$$= 49\ 393.38$$

Management Interpretation

The father will have R49 393.38 available to pay for his son's tertiary education in five years' time.

When compared to Example 16.13's future value (of R49 025.69) it can be seen that if the father made deposits at the *beginning* instead of at the end of each month, an additional R367.69 would have accrued to his account from the extra interest earned.

Present Value of an Ordinary Annuity Due

The *present value*, P_v, for an *ordinary annuity due* is given as:

$$P_v = R \frac{[1 - (1 + i)^{-n}](1 + i)}{i} \qquad \textbf{16.11}$$

Again, the additional term in the present value formula is $(1 + i)$, which accounts for the extra period's interest earned or owing.

Example 16.18 Retirement Annuity (Quarterly Compound Interest)

Mrs Bosman has won R75 000 in a competition. She decides to invest it in a retirement annuity, which will pay her a regular income *quarterly*, *in advance*, at 12% p.a. compounded quarterly for 10 years. She will receive her first income cheque the moment she pays her winnings of R75 000 to the Investment Company. How much will Mrs Bosman receive quarterly in advance for the next 10 years?

Solution

Since the income is received *quarterly in advance*, this retirement annuity is an *ordinary annuity due*. Also, since an initial lump sum is paid over, this refers to a *present value* amount, hence the *present value formula* for an ordinary annuity due must be used.

Required to find: the *regular quarterly income*, R, from the retirement annuity.

Given $P_v = 75\,000$ (the initial investment)

$i = 0.03$ (interest compounded quarterly, i.e. $\frac{0.12}{4} = 0.03$ per quarter)

$n = 40$ quarters (number of regular quarterly payment periods = 10 years × 4 = 40 quarters)

Using the present value Formula 16.11, solve for R:

$$75\,000 = R\frac{[1-(1+0.03)^{-40}](1+0.03)}{0.03}$$

$$75\,000 = R(23.80822)$$

$$R = \frac{75\,000}{23.80822}$$

$$= 3\,150.17$$

Management Interpretation

Mrs Bosman will receive R3 150.17 each quarter in advance for 10 years from her retirement annuity.

16.8 Deferred Annuities

A **deferred annuity** is an annuity for which the *first payment* of the series of regular payments will be made at *some future date*, rather than immediately.

Deferred annuity calculations involve finding either the present value, P_v, of the deferred annuity or the regular payments, R, made after the period of deferment. Both values can be found from the same *deferred annuity formula*, which is:

$$P_v = R\left(\frac{1-(1+i)^{-(m+n)}}{i} - \frac{1-(1+i)^{-m}}{i}\right) \qquad \textbf{16.12}$$

Where: m = number of deferred periods

n = number of payments

i = interest rate per period

R = regular payment amount per period

P_v = present value of the deferred annuity

The *rationale* of the formula is based on two principles:

- that an initial lump sum (present value) deposited in an investment account, for example, attracts *compound interest* for the duration of the *deferred period*
- this compounded future value, F_v, at the end of the deferred period, becomes the new present value, P_v, which is then paid out in regular instalments as an *ordinary annuity certain*.

The ordinary annuity certain formula for present value (Formula 16.9) is used to find these regular payment amounts, R.

Example 16.19 Pension Fund (Deferred Payment)

A R250 000 pension fund is set up for Agnes Mbeki for when she retires after three more years of service. The money in the fund must be paid out in five *equal annual* instalments to Agnes, starting on her retirement date (i.e. she will receive the first instalment on her retirement).

How much will Agnes receive from each instalment, where interest is compounded annually at 8% p.a.?

Solution

Required to find: the *regular payment amount*, R.

Since the first payment only takes place at the end of year 3, the annuity is deferred for two years. The annuity starts at the beginning of year 3 and payment occurs at the end of year 3.

Given	$P_v = 250\,000$	(the initial lump sum)
	$i = 0.08$	(the interest rate p.a.)
	$m = 2$	(the deferred number of periods)
	$n = 5$	(the number of payment periods)

Use the deferred annuity Formula 16.12 to find the regular payment amount, R:

$$250\,000 = R\left[\frac{(1-(1+0.08)^{-(2+5)})}{0.08} - \frac{(1-(1+0.08)^{-2})}{0.08}\right]$$

$$250\,000 = R[5.20637 - 1.783265]$$

$$250\,000 = R[3.423105]$$

$$R = \frac{250\,000}{3.423105}$$

$$= 73\,033.11$$

Management Interpretation

Agnes Mbeki will receive R73 033.11 at the end of three years from now, and each year thereafter for another four years.

Example 16.20 Education Trust Fund (Deferred Payment)

How much must be deposited in an education trust fund today for a schoolgirl such that she will receive R25 000 in four years' time, and an equal amount for *another* four years thereafter? Assume that interest is compounded annually at the rate of 10% p.a.

Solution

Required to find: the *present value*, P_v, of the education trust fund.

The annuity formula applies from the beginning of year 4, hence the annuity is deferred for three years. In addition, the schoolgirl will receive five equal payments from the annuity.

Given $R = 25\,000$ (the regular annual payment)
$i = 0.10$ (the interest rate p.a.)
$m = 3$ (the deferred number of periods)
$n = 5$ (the number of payment periods: at the end of year 4 and 4 further payments = 5)

Use the deferred annuity Formula 16.12 to find its present value, P_v.

$$P_v = 25\,000\left[\frac{(1-(1+0.1)^{-(3+5)})}{0.1} - \frac{(1-(1+0.1)^{-3})}{0.1}\right]$$
$$= 25\,000[5.334926 - 2.486852]$$
$$= 25\,000[2.848074]$$
$$= R71\,201.86$$

Management Interpretation

The education trust fund must be set up with an initial investment of R71 201.86 in order to pay a schoolgirl R25 000 for five years, starting in four years' time.

16.9 Application: Net Present Value Method

To evaluate the profitability of investment decisions for projects, the *time value* of money must be taken into account. This requires that all cash inflows (incomes) and cash outflows (payments) over time must be expressed in *present value* terms to allow for comparisons. This process is called *discounted cash flow analysis* (DCF).

The *net present value method* is a DCF approach and is used to assess the profitability of different project alternatives each with their own cash inflows and outflows over time.

The **net present value** (NPV) of a project is the difference between the *present value* of all cash *inflows* and the *present value* of all cash *outflows* over the duration of the project.

$$\text{NPV} = \text{PV(cash inflows)} - \text{PV(cash outflows)} \qquad 16.13$$

The *decision rule* to evaluate investment projects using the NPV approach is as follows:

- If NPV > 0, then a project is financially *viable* (profitable). This means that the PV of the cash inflows exceeds the PV of the cash outflows.
- If NPV = 0, then a project will *break even*. Here the PV of the cash inflows equals the PV of the cash outflows.
- If NPV < 0, then a project is financially *non-viable* (unprofitable). This means that the PV of the cash inflows is less than the PV of the cash outflows.

To apply the NPV method for project evaluation, a *cost of capital* must be assumed. The cost of capital is the *discount rate* that would be applied to each cash inflow and outflow over time. It represents the 'interest rate' that the company could earn on the money if it were invested in a savings account.

Example 16.21 New Machinery Evaluation (NPV Approach)

A manufacturing company wants to purchase a new metal stamping machine. Two options are available, machine A and machine B. The expected cash inflow from each machine over five years is shown in Table 16.2. Machines are scrapped after five years. Machine A will cost R228 000 and machine B will cost R344 000 to purchase. Assume a cost of capital of 10% p.a.

Table 16.2 Expected cash inflows for each machine over five years

Projected cash inflows from machine outputs		
Year	**Machine A**	**Machine B**
1	65 000	80 000
2	65 000	80 000
3	65 000	80 000
4	65 000	80 000
5	65 000	80 000
Total	**325 000**	**400 000**

Management Question

Which machine would be more profitable to purchase today?

Solution

The *present value* formula for *compound interest* calculations (Formula 16.4) is used to *discount* each machine's projected cash flow amount to the present period.

To illustrate, for machine A:

- the present value of year 1's cash flow is $\frac{65\,000}{(1.01)^1}$ = R59 091
- the present value of year 3's cash flow is $\frac{65\,000}{(1.01)^3}$ = R48 835
- the present value of year 5's cash flow is $\frac{65\,000}{(1.01)^5}$ = R40 360.

The present value calculation for all projected cash inflows (incomes) for both machines is shown in Table 16.3.

Table 16.3 Summary of PVs for each machine's cash inflows

Year	Machine A		Machine B	
	Projected cash flows	Present value	Projected cash flows	Present value
1	65 000	59 091	80 000	72 727
2	65 000	53 719	80 000	66 116
3	65 000	48 835	80 000	60 105
4	65 000	44 396	80 000	54 641
5	65 000	40 360	80 000	49 674
Total revenue	**325 000**	**246 401**	**400 000**	**303 263**
Machine cost		**228 000**		**344 000**
NPV		**18 401**		**–40 737**

Based on Table 16.3, the net present values (NPV) for each machine are:

Machine A: NPV = R246 401 – R228 000 = R18 401.

Machine B: NPV = R303 263 – R344 000 = –R40 737.

Management Interpretation

Since the NPV for machine A is positive (R18 401) while the NPV for machine B is negative (–R40 737), the management of the production company must purchase machine A. If they purchased machine B, they would incur a loss over the five-year life of the machine.

16.10 Using *Excel* (2013) for Financial Calculations

There are no *Excel* function keys for either simple interest or compound interest calculations of lump sum investments or borrowings. Function keys *are* available for the following financial calculations:

(a) Effective rate of interest: use **EFFECT(nominal_rate, Npery)**, where **Npery** is the number of compounding periods per year.

(b) Ordinary annuity – future value (FV): The function key **FV** can be used to compute the future value of both an ordinary annuity certain and an ordinary annuity due.

FV(rate, nper, pmt, pv, type)

(c) Ordinary annuity – present value (PV): The function key **PV** can be used to compute the present value of both an ordinary annuity certain and an ordinary annuity due.

PV(rate, nper, pmt, fv, type)

In (b) and (c), the following values are required:

- **rate** – interest rate for the compounding period (expressed as a decimal)
- **nper** – total number of payment periods
- **pmt** – the payment made each period
- **pv** – present value at the beginning of the investment (omit or set **pv** = 0)

- **fv** – future value at the end of the investment (omit or set **fv** = 0)
- **type** – represents the timing of the regular payments: if **type** = 1, payments are made at the *beginning* of each period (i.e. an ordinary annuity due calculation), if **type** = 0, payments are made at the *end* of each period (i.e. an ordinary annuity certain calculation).

16.11 Summary

This chapter dealt with two major areas of financial calculations, namely *interest* calculations and *annuity* calculations. A distinction was drawn between the two types of interest calculations, namely *simple interest* and *compound interest*. Calculations for each interest type require finding one of four values: the *present value*, the *future value*, the *rate of interest* or the *term* of an investment or loan. In compound interest calculations, it was emphasised that the interest rate and term must coincide with the conversion (or compounding) period.

This chapter divided annuities into *ordinary annuities* (for which payments begin immediately) and *deferred annuities* (for which payments commence only after an initial period of deferment). Ordinary annuities are further divided into two types: *ordinary annuities certain* and *ordinary annuities due*. They differ according to the commencement of payments within the first period: the former (ordinary annuities certain) commences payments at the end of the first period, while the latter (ordinary annuities due) starts payments at the beginning of the first period. Ordinary annuity calculations require finding one of three values: the *present value* of an annuity, the *future value* of an annuity or the *regular payments*. *Excel* offers the functions PV and FV to compute the present value and future values, respectively, of ordinary annuities.

The *net present value* (NPV) method of project evaluations was illustrated as an application of *compound interest* calculation. This approach is extensively used in practice to determine the financial profitability of alternative investment proposals.

Excel provides function keys only for *effective rates of interest* calculations and *ordinary annuity* (both certain and due) calculations.

Exercises

1 Explain the difference between simple interest and compound interest.

2 Is a lump sum investment based on simple interest likely to have a higher maturity value (F_v) than a lump sum investment based on compound interest for the same lump sum and term? Explain.

3 Will a lump sum investment that is compounded quarterly have a higher maturity value (F_v) than a lump sum investment that is compounded annually, assuming the same rate of interest and term? Explain.

4 What is the difference between the nominal rate of interest and the effective rate of interest on a lump sum investment?

5 Explain the term ‘annuity’.

6 Explain the difference between an ordinary annuity and a deferred annuity.

7 Explain the difference between an ordinary annuity certain and an ordinary annuity due.

8 Explain the term ‘net present value’. In what way is this method used in practice?

9 An investor deposits R15 000 into a fixed deposit account that pays 8% p.a. The investment is for five years.
- (a) What is the maturity value of the deposit if simple interest is paid?
- (b) What is the maturity value of the deposit if interest is compounded annually?
- (c) What is the maturity value of the deposit if interest is compounded half-yearly?

10 A couple decides to save to buy a car in two years' time. They will invest a fixed sum into a two-year fixed deposit savings account that pays interest at 12% p.a. How much would they need to invest today to have R150 000 at the end of the two-year period if:
- (a) interest is paid annually?
- (b) interest is paid half-yearly?
- (c) interest is paid monthly?

11 How long will it take a sum of money to treble:
- (a) at 16% p.a. simple interest?
- (b) at 16% p.a. compound interest, with interest paid annually?
- (c) at 16% p.a. compounded quarterly?

12 What amount of money must be invested now so as to accumulate to R10 525 in 30 months' time at:
- (a) 14% per annum simple interest?
- (b) 14% per annum interest, compounded annually?

13 An investment account is advertised as offering an interest rate of 15% p.a. compounded monthly.
- (a) Find the effective interest rate per annum.
- (b) Use the *Excel* function **EFFECT** to find the effective rate.

14 The interest rate for an investment is 9% per annum compounded quarterly.
- (a) What is the effective interest rate per annum?
- (b) Use the *Excel* function **EFFECT** to find the effective rate.

15 An amount of R25 000 is invested at an interest rate of 11% per annum compounded half-yearly for n years. Find the value of n if at the end of this period the investment has accumulated to R58 890.

16 R2 000 is invested at 10% per annum compounded half-yearly. After three months, the interest rate changes to 12% per annum compounded monthly. Find the value of the investment after two years.

17 An investment of R7 500 grows to an amount of R10 200 where interest is compounded quarterly over three years. What is the annual rate of interest?

18 Find the interest rate required for an investment of R5 000 to grow to R8 000 in four years if interest is compounded monthly.

19 How much must an investor deposit today to withdraw R25 000 after two years and nine months from an investment that pays interest at 9% p.a. compounded quarterly?

20 If R21 353.40 is invested at 12% p.a. compounded annually, how long will it take for the capital plus the interest to amount to R30 000?

21 If an invested amount is to double in seven years, find the nominal interest rate at which it must be invested if interest is compounded quarterly.

22 A monthly deposit of R1 600 is made at the end of each month into an account at an interest rate of 12% p.a. compounded monthly.

(a) How much is in the account immediately after the 15^{th} monthly deposit of R1 600?
(b) Use the appropriate *Excel* function (FV or PV) to compute (a).
(c) Re-calculate (a) and (b), assuming that the monthly deposit takes place at the beginning of each month.

Note: The final deposit will take place at the beginning of month 15.

23 When a sum of money is invested for a period of nine years at an interest rate of 7% per annum, the difference between the interest earned if calculated using the compound interest method (compounded annually) and the simple interest method is R334.16. What was the capital sum of the money invested?

24 A nurse wants to buy a new small car in three years' time. The car she wants to buy currently costs R80 000 but is expected to escalate at a compound rate of 4% per annum. The nurse can invest money at 9% p.a. compounded monthly.
(a) How much would she need to invest at the *end* of each month to purchase the car after three years?
(b) How much would she need to invest at the *beginning* of each month to purchase the car after three years?

25 A debt of R8 500 is to be settled in equal monthly instalments over three years beginning at the *end* of the first month. Interest of 18% p.a. compounded monthly is paid on the debt.
(a) How much must be paid each month to settle the debt?
(b) How much interest was paid over the period of the debt? What percentage of the original debt does this represent?

26 On reaching the age of 60, an employee of Tramcor (Pty) Ltd has the option of receiving a pension of R8 750 per month for five years, payable at the *end* of each month or taking an equivalent lump sum gratuity on retirement. If an employee who will retire soon decides to take the gratuity, how much will she receive? Assume interest is compounded monthly at 10% p.a.

27 Mrs Mabula pays a quarterly premium *in advance* of R750 on an endowment policy with Sun Life Insurance. The term of the policy is 15 years.
(a) How much would she receive on maturity, assuming an interest rate of 14.5% p.a. compounded quarterly?
(b) Use an appropriate *Excel* function (PV or FV) to compute (a).

28 To save for the future education of their child, a couple is saving R540 at the *end* of every month which they invest in a savings account that carries interest of 12% p.a. compounded monthly. After 24 deposits they increase the payments to R750 at the end of every month.

How much money will be available at the end of nine years from the day they first started to save? (Assume that no withdrawals were made.)

29 An investment analyst has told you that it is better to invest R1 000 at the *end* of every month for one year in a scheme that pays interest of 8.5% p.a. compounded monthly, than to invest R3 000 at the *end* of every quarter in a scheme paying 10% p.a. compounded quarterly over the same period of time.
(a) Would you take his advice and how would you justify your decision? Show all calculations.

(b) Use the appropriate *Excel* function (PV or FV) to answer (a).
(c) If each investment took place at the *beginning* of each month, would the advice still be valid? Justify using the appropriate *Excel* function (PV or FV). Explain your answer.

30 An advertisement for a new 1 600cc motor vehicle offers the following deal: a deposit of R20 000 and 48 monthly payments in arrears (i.e. at the *end* of each month) of R2 200 at an interest rate of 'prime plus 1%' p. a. compounded monthly.

The prime interest rate is currently 8% p.a. The rate of interest will be fixed for the duration of the purchase contract.

(a) Calculate the purchase price of the motor vehicle.
(b) Use an appropriate *Excel* function (PV or FV) to compute (a).

31 A couple, who are both working, each invest R500 at the end of each month in a joint savings account that carries interest of 8% p.a. compounded monthly. After two years the interest rate rises to 10% p.a. compounded monthly, and the couple continues to pay into the account for a further year.

(a) How much money will be in the account at the end of this period? Assume that no withdrawals were made.
(b) Use an appropriate *Excel* function (PV or FV) to compute (a).

32 (a) Find the amount in an account after 12 monthly payments of R200 have been made, if R300 was withdrawn from the account after five months (i.e. at the time of the fifth R200 payment) and again after 10 months (i.e. at the time of the 10^{th} R200 payment). Interest is calculated at 12% p.a. compounded monthly.
(b) Use either a calculator or an appropriate *Excel* function to assist in the calculation of the amount at the end of the 12-month period.

33 A student loan of R26 000 is to be repaid by 12 equal quarterly instalments at an interest rate of 14% p.a. compounded quarterly.

(a) What is the amount of each instalment if each instalment is only paid at the *end* of each quarter?
(b) What is the amount of each instalment if each instalment must be paid at the *beginning* of each quarter?
(c) If a student is free to select whether repayments occur at the beginning or at the end of each quarter, which option should be chosen? Why? (Assume funds are available for either option.)

34 A businessman borrows R30 000 to start a laundry business. The loan must be repaid in 24 equal monthly repayments. The terms of the loan are that the repayments are due only at the *end of each month* and that the first repayment is due only *after* three years when the business is assumed to be established.

What is the amount of each repayment, assuming an interest rate of 16% p.a. compounded monthly?

35 If an investment of R18 000 has grown to R40 697.70 over five years for which interest is compounded half-yearly, what is the nominal rate of interest per annum on the investment?

36 A house owner decides to save to build an extension to her house. She plans to save R2 000 at the end of each month and place it into an investment account that pays a fixed interest rate of 12% p.a. compounded monthly. How long will it take the house owner to save R103 757.98?

37 X16.37 – investment options

An investor has R60 000 to invest. He has an option of buying a share in an established long-haul trucking business that transports goods up the West Coast. Alternatively, he can invest his funds into a newly launched laundry business. Assume a cost of capital of 12% p.a. The projected cash flows for the next five years are as follows:

		Investment options	
		Trucking	**Laundry**
Initial investment (R)		60 000	60 000
Annual cash flow (R)	Year 1	32 000	0
	Year 2	38 500	7 500
	Year 3	26 000	45 000
	Year 4	13 000	37 500
	Year 5	9 500	55 500

Based on the projected cash-flow values for each investment option, which option should the investor choose? Use the NPV method.

Solutions to Exercises

Chapter 16

16.1 See text.

16.2 No – a compounded amount will earn more interest than a simple interest investment.

16.3 Yes – quarterly compounding will result in interest being capitalised sooner.

16.4 – 16.8 See text.

16.9 (a) F_v = R21 000
(b) F_v = R22 039.92
(c) F_v = R22 203.66

16.10 (a) P_v = R119 579.10
(b) P_v = R118 814
(c) P_v = R118 134.90

16.11 (a) n = 12.5 years
(b) n = 7.402 years
(c) n = 7.003 years (28.011 quarters)

16.12 (a) P_v = R7 796.30
(b) P_v = R7 585.08

16.13 (a) r = 16.0755% p.a.
(b) **=EFFECT(0.16.12)** = 0.160755 (16.0755%)

16.14 (a) r = 9.3083% p.a.
(b) **=EFFECT(0.09,4)** = 0.093083 (9.3083%)

16.15 n = 8.0013 years

16.16 F_v = R2 525.65

16.17 i = 10.3819% p.a.

16.18 i = 11.8078% p.a.

16.19 P_v = R19 572.37

16.20 n = 3 years

16.21 i = 10.0257% p.a.

16.22 (a) F_v = R25 755.03
(b) =FV(0.01,15,1600) gives R25 755.03
(c) F_v = R26 012.58
(d) =FV(0.01,15,1600,1) gives R26 012.58

16.23 P_v = R1 603

16.24 (a) R (regular payment) = R2 186.71 (deposit at *end* of month)
(b) R (regular payment) = R2 170.43 (deposit at *beginning* of month)

16.25 (a) R (monthly payment – at month end) = R307.30
(b) Interest paid = R2 562.63; 30.1486%

16.26 P_v = R411 821.98

16.27 (a) F_v = R160 149.71
(b) **=FV(0.03625,60,750,1)**

16.28 R131 603.17

16.29 (a) At *end* of period
Monthly: F_v = R12 478.72
Quarterly: F_v = R12 457.55
Monthly investment scheme is better.
(b) *Excel* functions
Monthly: **=FV(0.0070833,12,1000)**
Quarterly: **=FV(0.025,4,3000)**
(c) At *beginning* of period
Monthly:
FV(0.0070833,12,1000,1) = R12 567.11
Quarterly: **FV(0.025,4,3000,1)** = R12 768.99
Quarterly investment scheme is better.

16.30 (a) Purchase price = R88 406.52 + R20 000 = R108 406.52
(b) *Excel* function
Monthly: **=PV(0.0075,48,2200)** = R88 406.52

16.31 (a) F_v = R41 214.30
(b) For first two years:
=FV(0.006667,24,1000) = R25 933.19
Compounded for one year = 25 933.19(1.008333)12 = R28 648.73
For year 3: = **FV(0.008333,12,1000)** = R12 565.57
Total funds available = R28 648.73 + R12 565.57 = R41 214.30

16.32 (a) F_v = R1 908.83
(b) For first five months:
=FV(0.01,5,200) = R1 020.20
Compounded for seven months:
$720.20(1.01)^7$ = R772.15
For months 6 to 10:
=FV(0.01,5,200) = R1 020.20
Compounded for two months:
$720.20(1.01)^2$ = R734.68
For remaining two months:
=FV(0.01,2,200) = R402.00
Balance at end of 12 months = R772.15 + R734.68 + R402 = R1 908.83

16.33 (a) R(repayment at end of each quarter) = R2 690.58
(b) R(repayment at beginning of each quarter) = R2 599.60
(c) Choose to repay at beginning of a quarter as total repayment is less.

16.34 R(repayment beginning after three years) = R2 366.32

16.35 Nominal interest rate: i = 17% p.a.

16.36 n = 42 months (three years and six months)

16.37 NPV(trucking) = R31 421.97
NPV(laundry) = R33 333.18
Recommend laundry investment.

FINANCIAL CALCULATIONS

Simple interest $F_v = P_v\,(1 + in)$ **16.1**

Compound interest $F_v = P_v\,(1 + i)^n$ **16.2**

$F_v = P_v(1 + \frac{i}{k})^{nk}$ **16.3**

where k = number of compounding periods in a year.

Effective rate of interest $r = \left(1 + \frac{i}{m}\right)^m - 1$ **16.7**

Ordinary annuity certain $F_v = R\frac{(1 + i)^n - 1}{i}$ **16.8**

$P_v = R\frac{1 - (1 + i)^{-n}}{i}$ **16.9**

Ordinary annuity due $F_v = R\frac{[(1 + i)^n - 1](1 + i)}{i}$ **16.10**

$P_v = R\frac{[1 - (1 + i)^{-n}](1 + i)}{i}$ **16.11**

Deferred annuity $P_v = R\left(\frac{1 - (1 + i)^{-(m+n)}}{i} - \frac{1 - (1 + i)^{-m}}{i}\right)$ **16.12**