Example D: Barnes (2007)

The Relationship between Mathematics Subject Matter Knowledge and Classroom Practice

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1 Introduction

How does one mathematically determine whether the gradient of a straight line is positive or negative? I asked this of a mathematics student teacher I was observing and was surprised that he could provide no mathematical explanation. Instead, he explained that a positive gradient could be recognised by the fact that if you were walking along the line, it would be like walking up a mountain, so you would feel really positive. On the other hand, the negative gradient or slope is like coming down a mountain and one usually feels negative coming down a mountain. He confessed that he relied mainly on memorisation to explain mathematical concepts.

This is one of many similar examples where mathematics is endorsed as a process of rote memorisation rather than discipline requiring understanding. During my role as a mathematics methodologist (or specialisation lecturer), I became increasingly frustrated and concerned at the content knowledge as well as teaching and learning strategies being demonstrated by pre-service mathematics students during such practical teaching periods. Despite the global reform being initiated in mathematics education, the students continued to demonstrate a traditional and rote learning approach to teaching mathematics, with only superficial gestures towards a more constructivist paradigm. With their own experiences of mathematics teaching at school most likely being limited to a traditional approach and the lack of deep change occurring in most schools they would end up teaching in, I began to wonder how we can most effectively achieve the change in pedagogy we are aiming towards. And how much of this may be dependent on the mathematical content knowledge of teachers?

Along with many other countries, South Africa has experienced a radical curriculum reform during the past ten years. Our latest curriculum based on a philosophy of Outcomes-Based Education (OBE) demands a range of teaching strategies and roles on the part of the teacher as outlined in the Norms and standards for educators (Department of Education [DoE], 2003). These include being mediators of learning, interpreters and designers of learning programmes and materials, leaders, administrators and managers, scholars, researchers and lifelong learners, community members, citizens and pastors, assessors and Learning Area or Phase specialists. The curriculum statement also makes the point that setting and achieving outcomes encourages a learner-centred and activity-based approach to education.

This reform in the type of teacher envisioned has also brought about changes in pre-service teacher training programmes. Much of
the research focusing on teacher training is making an attempt to find out how training should be tailored in order to optimally prepare students and teachers for the changing role of teaching they have to fulfill (e.g. Adler, Davis & Kazima, 2005; Ball, 1990; Ma, 1999; Peressini, Borko, Romagnano, Knuth & Willis, 2004; Shulman, 1986, 1987). The aim of this research project is to contribute to the existing body of research in this regard, by investigating the relationship between the subject matter knowledge of secondary pre-service mathematics teachers and the way in which they teach the subject.

2 Background to the research

The intellectual puzzle I engage with in this research project has emerged out of a variety of experiences I have had over the last few years in my position as a lecturer training pre-service mathematics teachers. The experiences involve workshops, lectures, interviews, observations and general interactions I have had with both pre-service and current teachers. The problem is encapsulated in the limited conceptual understanding of mathematics demonstrated by teachers and students of mathematics, the poor performance of learners in South Africa in mathematics and the possibility of improving learner performance with improved mathematical understanding on the part of the teachers. In the sections that follow, further insight into these experiences and the problem is provided.

2.1 Training of pre-service mathematics students

Students in a third-year methods class were asked if the calculation in Figure 1 could be performed by dividing the numerators and then dividing the denominators.

\[
\frac{21}{35} \div \frac{3}{7} = \frac{7}{5}
\]

Figure 1. Division of fractions calculation

The immediate response of most of the class was a resounding “no”. After doing the calculation their own way though (e.g., see Figure 2), most of the students then noted that the answer was in fact correct. At least half the class were still adamant, however, that the calculation could not be done using the technique first mentioned, even though the answer was correct. When asked to write down why they thought it could not be done that way, the general response was that “we were not taught to do it that way”.

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Students were then further requested to indicate how they would approach teaching the topic of division of fractions to a class. All the students focused their approach on teaching learners to multiply by the reciprocal. Without exception, not one of the students could produce a mathematically correct reason for why the method they were proposing to teach learners is acceptable and why it works. The most common reason they gave was that division and multiplication are inverse operations and that the second fraction should therefore be inverted. When confronted with the counterexample of applying their conjecture to the addition and subtraction of fractions, although aware of the incorrectness in their thinking, students were unable to find a suitable mathematical reason for why we multiply by the reciprocal instead of dividing fractions.

This is one of many available vignettes providing anecdotal evidence of how students demonstrated their lack of conceptual understanding and their limited, instrumentalist view of mathematics, which Ernest (1988, p. 10) proposes:

... is the view that mathematics, like a bag of tools, is made up of an accumulation of facts, rules and skills to be used by the trained artisan skilfully in the pursuance of some external end. Thus mathematics is a set of unrelated by utilitarian rules and facts.

I increasingly became concerned about students who may continue to hold this view of mathematics as they themselves enter the teaching profession. How would this view of mathematics enable them to be effective “mediators of learning” and “Learning Area specialists” as required by the norms and standards set out for educators (DoE, 2002, p. 3)? And would this view and lack of insight perhaps confine them to a more traditional approach to teaching mathematics in their pedagogy?
2.2 **South African performance in mathematics**

South Africa took part in the Third International Mathematics and Science Study (TIMSS – now referred to as Trends in Mathematics and Science Study) in 1995, 1999 and 2003, of which the latter two were conducted on Grade 8 learners. On all three occasions, South Africa was placed in the last position (in 2003, out of approximately 50 countries), being outperformed by other African countries such as Botswana, Tunisia, Egypt and Morocco (Howie, 2002; Reddy, 2006). TIMSS made use of Item Response Theory to calculate the achievement scores, with a scale of 800 points and a standard deviation of 100 points. In the 2003 results, the average scale score for Grade 8 South African learners was 264 (SE = 5.5), which was significantly lower than the international average scale score of 467 (SE = 0.5). The average scale score of South Africa in the 2003 TIMSS study compared to the average scale scores of other African countries that took part is depicted in Table 1 below. What is interesting to note on other statistics provided in the report is that out of all these countries below, South Africa has the second highest gross national income per capita out of the group (Reddy, 2006).

**Table 1: Comparison of South Africa average mathematics scale score in TIMSS 2003 with other African countries**

<table>
<thead>
<tr>
<th>Country</th>
<th>Average age of learner</th>
<th>Average scale score</th>
</tr>
</thead>
<tbody>
<tr>
<td>South Africa</td>
<td>15.1</td>
<td>264</td>
</tr>
<tr>
<td>Botswana</td>
<td>15.1</td>
<td>366</td>
</tr>
<tr>
<td>Tunisia</td>
<td>14.8</td>
<td>410</td>
</tr>
<tr>
<td>Ghana</td>
<td>15.5</td>
<td>276</td>
</tr>
<tr>
<td>Jordan</td>
<td>13.9</td>
<td>424</td>
</tr>
<tr>
<td>Egypt</td>
<td>14.4</td>
<td>406</td>
</tr>
<tr>
<td>Morocco</td>
<td>15.2</td>
<td>387</td>
</tr>
</tbody>
</table>

Looking at results of regional and then national testing does not enhance the picture. UNICEF and UNESCO collaborated to conduct a study called Monitoring Learner Achievement (MLA) in numeracy, literacy and life skills. South Africa took part in the study in 1999 (Grade 4 learners) and in 2003 (Grade 6 learners), along with other African countries. The Grade 4 numeracy results of 1999 are depicted in Table 2 (see Chinapah, 2000; Strauss & Burger, 2000; Human Sciences Research Council, 2007).
Once again, South Africa achieved the lowest average score (30%) compared to other African countries. The results of the literacy and life skills averages have also been included in Table 3 for comparative purposes. It is interesting here to note that while South African learners outperformed, for example, Niger in literacy and life skills, our numeracy average was still significantly lower than that of their learners.

Table 2: Results of Grade 4 MLA study for numeracy, literacy and life skills

<table>
<thead>
<tr>
<th>Country</th>
<th>Numeracy average</th>
<th>Literacy average</th>
<th>Life skills average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tunisia</td>
<td>60.4</td>
<td>77.9</td>
<td>74.7</td>
</tr>
<tr>
<td>Mauritius</td>
<td>58.5</td>
<td>61</td>
<td>58</td>
</tr>
<tr>
<td>Morocco</td>
<td>56.4</td>
<td>67.6</td>
<td>62.3</td>
</tr>
<tr>
<td>Botswana</td>
<td>51</td>
<td>48</td>
<td>56</td>
</tr>
<tr>
<td>Uganda</td>
<td>49.3</td>
<td>58.7</td>
<td>66.8</td>
</tr>
<tr>
<td>Madagascar</td>
<td>43.7</td>
<td>54.7</td>
<td>72.1</td>
</tr>
<tr>
<td>Mali</td>
<td>43.6</td>
<td>51.8</td>
<td>56.9</td>
</tr>
<tr>
<td>Malawi</td>
<td>43</td>
<td>35</td>
<td>77</td>
</tr>
<tr>
<td>Senegal</td>
<td>39.7</td>
<td>48.9</td>
<td>45.7</td>
</tr>
<tr>
<td>Niger</td>
<td>37.3</td>
<td>41.1</td>
<td>44.7</td>
</tr>
<tr>
<td>Zambia</td>
<td>36</td>
<td>43</td>
<td>51</td>
</tr>
<tr>
<td>South Africa</td>
<td>30.2</td>
<td>48.1</td>
<td>47.1</td>
</tr>
</tbody>
</table>
If one looks at Table 3, which is a breakdown of the mathematics enrolment and performance at Senior Certificate level from 1996 to 2002,¹ the national performance in terms of the mathematics achievement of our learners at school-leaving level is also concerning.

Table 3: National Senior Certificate Examination results (1996–2002)

<table>
<thead>
<tr>
<th></th>
<th>1996</th>
<th>1999</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of candidates</td>
<td>518 225</td>
<td>511 225</td>
<td>489 941</td>
<td>449 371</td>
<td>444 259</td>
</tr>
<tr>
<td>Percent writing maths</td>
<td>42%</td>
<td>55%</td>
<td>58%</td>
<td>59%</td>
<td>59%</td>
</tr>
<tr>
<td>Percent passing maths</td>
<td>21%</td>
<td>24%</td>
<td>26%</td>
<td>27%</td>
<td>32%</td>
</tr>
<tr>
<td>Pass on SG</td>
<td>17%</td>
<td>20%</td>
<td>21%</td>
<td>23%</td>
<td>27%</td>
</tr>
<tr>
<td>Pass on HG</td>
<td>4%</td>
<td>4%</td>
<td>5%</td>
<td>4%</td>
<td>5%</td>
</tr>
</tbody>
</table>

SG: Standard Grade
HG: Higher Grade

According to the above results, our learners are underperforming in mathematics both nationally and internationally. Studies have been done to explore why this is the case, where factors that contribute to mathematics performance have been analysed (Howie, 2002; Reddy, 2006). It is not my aim to explore these further here, but my underlying belief is that teaching must have an enormous impact on our learners’ performance. Surely, if our teaching improves, so should the quality of our learning?

Howie (2002) analysing data collected from teachers in the TIMSS 1999 study found that in South African classrooms, significantly more time (21%) was spent on reteaching and clarification of content or procedures than in other countries on average (13%). South African teachers also spent more time on administrative tasks (13% compared to average 5% in other countries) and reviewing homework (26%) compared to the average of other countries (12%). Regarding pedagogical practices, teachers of 16% of South African learners placed a high emphasis on mathematics reasoning and problem solving, which was comparable with the international

¹ Data up to 2006 will be accessed for the actual dissertation. At date of writing this proposal I had only managed to access data up to 2002.
average. However, while the pattern internationally appeared to be that learners of teachers who claimed to have this approach would achieve a corresponding higher achievement, this was not the case with South African mathematics. In fact, the opposite was true. South African learners whose teachers reported placing a high emphasis on reasoning and problem solving achieved lower results (260 points) compared to learners whose teachers placed a lower emphasis on this approach (303 points).

Reddy (2006) compared the results of the 1999 and 2003 TIMSS data to find that on average the scores had decreased, although the difference was not statistically significant. She makes the following comment in her report (p. 52):

Since 1998 (with the introduction of C2005), there have been many professional development courses and programmes for teachers. In addition, numerous interventions by government, private sector, business and non-governmental organisations have been made in schools, especially the African schools, with the objective of improving the state of mathematics and science education. However, it seems that despite these programmes there has been a decrease in mathematics performance in many schools.

Perhaps it is time to start asking ourselves why our professional development courses (and I would also venture to include pre-service teacher training programmes here) are not having the desired improved effect on the mathematics performance of our learners. Are we perhaps expecting teachers to change their pedagogical beliefs and practices, when in fact their subject matter knowledge is a limiting factor in enabling them to effectively do this? I hope to unravel and understand more of this within the context of this investigation on pre-service teachers.

### 2.3 **Contract research project**

During the course of 2004 I managed an independent evaluation for a non-governmental organisation (NGO)\(^2\) in the form of contract research. The task was to evaluate the impact/effectiveness of an intervention it was funding. The particular intervention was aimed at additional training and support for Intermediate Phase educators, mostly in rural areas. The funders were mainly interested in the impact of the intervention on the learners’ performance in the selected schools. The evaluation was carried out in a randomly selected sample of 12 schools in one South African province. As already mentioned, the evaluation sought to examine the impact

\(^2\) Permission in the process of being obtained from the NGO to make use of the data from the evaluation.
and effect of the intervention on firstly the educators (at which the intervention is primarily aimed) and as a more distant outcome, the learner performance.

The evaluation collected data from approximately 1 104 learners and an average of 17 educators from the 12 schools. A total of three instruments were used in collecting data from educators. These included semi-structured interviews (that were recorded and transcribed), observation schedules (completed by fieldworkers sitting in on lessons) and educator questionnaires that the educators involved completed. Learner performance was measured through the administration of a pre- and post-test, which were identical. Once completed, the tests were manually coded (marked) by fieldworkers and the data captured by data typists. A team from our centre consisting of a statistician and two researchers specialising in mathematics education analysed and interpreted the data.

During the semi-structured interview, educators were asked by a fieldworker to offer their definition and views of mathematics as a subject. This was done in order to ascertain how the educators valued the place of the subject in the curriculum and how confident they were in teaching it. An educator’s view of mathematics is also often an indicator of the way they are likely to teach it. To quote John Dussey (1992): ‘The conception of mathematics held by the educator may have a great deal to do with the way in which mathematics is characterized in classroom teaching’ (p. 42). Hersh (1986) makes the same point: ‘One’s conceptions of what mathematics is, affects one’s conception of how it should be presented. One’s manner of presenting it is an indication of what one believes to be most essential in it’ (p. 13).

Most of the educators that were interviewed held positive views of mathematics and claimed that it was a very necessary part of the curriculum. Quotes from the educators, such as those included below substantiate this:

I would say it's a very lovely subject, what is important is we are doing maths everyday of our lives. You go to the bus you pay, that's maths, you look at the watch, you go to the shop you buy that's maths. We are doing maths unconsciously, so maths is the subject to be taught everywhere.

I would say it's the mother of all the subjects because even if you didn't go to school but maths is always there, even if you can't read or write but some other people are able to calculate their money, they are able to say I want 1kg bag of rice or I want 10kg, that is maths, it's the most important subject, whether you like it or not but you are doing it anyway, unconsciously.

However, some also admitted they find it difficult and challenging to teach, but that they are “trying to rub all those stereotypes” that learners and educators often attach to the subject. Some of the
educators felt they are succeeding in this now that they had learnt ways to make more use of practical resources in their teaching, through the course of the intervention. Others voiced their continued fear and concerns of the subject:

... but although we are not good on it but we love it.
Because even our learners they are so difficult to grasp, so you don't know whether it's language or what.
You can say that maths is an interesting subject, but we, including our kids we are afraid of it.

The definitions of mathematics provided by the educators pertained mainly to the use of the subject as it relates to figures and the four basic operations (addition, subtraction, multiplication and division) as used in our daily lives:

But if I can define it [mathematics], with the knowledge I've got – I can say mathematics is measurement, because everything you measure is mathematics included. It can be information because you can get information from the radio, bearing in mind that it's four o'clock now, it's use, so now I'm using a watch through mathematics. There are so many things that I can say about defining, it can be measurement, I can say the distance, the counting ... learners can count, they can count change, when they get into the bus they must know the bus here from to town it's R10, it's R9.50, so they must know if I gave them R10 they must know that there's 50 cents change. So that is how mathematics works to me.

Maths is a subject dealing with numbers and measurements. It is used daily in our lives e.g. when buying groceries.

Only one of the educators alluded to it in the sense of “problem-solving” and another to the benefits of mathematics in improving the thinking of learners:

... maths to me as a whole it, is dealing with problem solving. It's true, the main concept of maths is to solve the problem.
... just in short I can say – I would say mathematics creates fast thinking in our pupils, they think very fast. So they will think very fast.

Data collected from the interviews were supported by observations from the fieldworkers who observed the educators actually teaching lessons. Out of the 25 classes observed, most of the educators explained the work by means of showing the learners examples. In 16 of these lessons, the educators used examples relating to real-life situations, while in 15 of the classes fieldworkers also observed learners solving contextual problems relating to their lives. However, only seven of the classes showed learners having the opportunity to negotiate meaning through discussing their understanding of concepts and strategies for solving problems with each other and the educator. Also relating to this, learners posing
problems to their educator and to each other was only observed in six of the lessons.

What can be concluded from the analysis done in relation to the educators' views and definitions of mathematics is that although all of them believe it is an important and worthwhile subject, they are not all very comfortable or confident teaching it. This could be due to a limited level of content knowledge as depicted in many of the definitions offered by the educators of what mathematics is. What does appear to be coming through, though, is that educators are making an effort to teach the subject in a very practical manner and to make it as relevant as possible to the daily lives of learners.

While it was encouraging to see educators moving towards a more practical approach to teaching mathematics, it concerned me that this encompasses only a small part of the scope of mathematics as a subject. In the Revised National Curriculum Statement (RNCS) the Department of Education provides the following definition for mathematics (DoE, 2002, p. 4):

Mathematics is a human activity that involves observing, representing and investigating patterns and quantitative relationships in physical and social phenomena and between mathematical objects themselves. Through this process, new mathematical ideas and insights are developed. Mathematics uses its own specialised language that involves symbols and notations for describing numerical, geometric and graphical relationships. Mathematical ideas and concepts build on one another to create a coherent structure. Mathematics is a product of investigation by different cultures – a purposeful activity in the context of social, political and economic goals and constraints.

It is my understanding that this definition, as well as the purpose, unique features and scope of mathematics as provided in the RNCS (DoE, 2002) is calling for more than a greater emphasis on a practical approach to teaching mathematics. The definition and purpose require educators to apply a range of teaching and learning strategies so that learners can gain the full benefit of mathematics. I therefore began to question what it is that either enables or limits educators from being more flexible in the range of teaching and learning strategies they apply in their classrooms. Reflecting on the data obtained from the evaluation outlined above, I noticed that this particular sample of educators did not seem resistant to making changes in their teaching strategies. They also felt that the resources and training provided during the course of the intervention had enabled them to be more practical in their teaching. I decided to further analyse the definitions teachers provided for mathematics to gain further insight into their conceptions and knowledge of mathematics. As the literature suggests (Ball, 1990; Ma, 1999), teachers’ conceptions and understanding of what mathematics is
could be a factor limiting the optimisation of a broader range of teaching and learning strategies within their classrooms. Many of the educators interviewed emphasised the practical day-to-day uses of mathematics when stating their definitions for the learning area. Classroom observations provided evidence of a greater emphasis on this practical aspect in their teaching. I therefore began to wonder if a relationship exists among educators’ knowledge of mathematics, how they acquire this knowledge, and how this manifests in relation to the range of teaching and learning strategies they employ in their classrooms.

This possible relationship, the poor performance of our learners in mathematics and my experiences with my own pre-service students led me to the literature on content knowledge in the teaching of mathematics, which is further expanded on in the following section.

2.4 Consulting the literature

To begin the process of searching for relevant literature on the content knowledge of mathematics teachers, I used a combination of the following keywords – content knowledge; mathematics; education; pre-service; student teachers – and initiated a search on various Internet search engines and academic databases. This led me to a paper entitled “Developing measures of teachers’ mathematics knowledge for teaching” by Hill, Schilling and Ball (2004). The article contains an overview of the original literature on content knowledge for teaching which was most helpful on setting me off on a literature “trail”.

The literature trail took me to a plethora of work, mostly by Ball (1989; 1990; 1991) on mathematics knowledge for teaching. Ball and her colleagues draw on Shulman’s contribution (1986) of pedagogical content knowledge, as well as the well-known work of Ma (1999). Other authors, such as Grossman, Wilson and Shulman (1990) and Leinhardt and Smith (1985) are also regarded as experts on the research in this regard.

Through the literature trail, it became evident that the term “content knowledge” is generally accepted as being more loaded than one’s knowledge of mathematical content. Shulman (1986), for example, distinguishes three categories of content knowledge: subject matter content knowledge, pedagogical content knowledge and curricular knowledge. Ball (1990) differentiates between the execution of a mathematical operation and the teacher’s ability to represent that operation accurately for learners. She therefore coined the terms “knowledge of mathematics” and “knowledge for mathematics”. Her later work, supported by other researchers such as Hill and Bass, attempts to identify, measure and address the mathematics knowledge
necessary for teaching. Leinhardt and Smith (1985) suggest that the most important two distinctions we should make regarding content knowledge of teachers relate firstly to their lesson structure knowledge and secondly to their subject matter knowledge. Grossman et al (1990), on the other hand, extended the number of categories to four. They suggest subject matter knowledge, general pedagogical content knowledge, pedagogical content knowledge and knowledge of the context. While Ma (1999) in her research did not define categories, she studied the “profound understanding of fundamental mathematics” (Hill, Shilling & Ball, 2004, p. 2) in order to compare the subject matter knowledge of elementary mathematics teachers in the United States and China. In a current research project in South Africa known as the Quantum Project (Adler, Davis, Kazima, Parker & Webb, 2005), Adler and her colleagues investigate and attempt to describe mathematics for teaching within an in-service training context. Their project is focused on middle and senior school mathematics teachers, foregrounding what mathematics they need to know and their knowledge of how to use this mathematics in order to teach mathematics effectively in diverse classroom contexts.

Reflecting on my puzzle through the lens I had now constructed from the literature, I decided that the term “subject matter knowledge” was most appropriate for the particular input that I wish to investigate. It is central to all the findings emerging from the “content knowledge” literature and depicts the specific construct I wish to more closely examine. My aim is therefore to study the classroom practice of pre-service secondary mathematics teachers in order to ascertain how their subject matter knowledge manifests in their classroom practice.

Having provided the background to this study, the actual problem, rationale and research questions are now discussed.

3 Problem statement

This study seeks to investigate the relationship between the subject matter knowledge of secondary pre-service mathematics teachers and how they teach mathematics.

3.1 Rationale

In South Africa, the last ten years have been full of an all-embracing educational reform initiative that was conceived after the demise of apartheid. After South Africa’s first democratic elections in 1994, a huge national project was embarked on and educational reform has been a central part of this reconstruction. This educational reform
has been driven by two imperatives: firstly, the need to overcome the
damage done by apartheid, and provide a system of education that
builds democracy, human dignity, equality and social justice, and
secondly, to establish a system of lifelong learning (DoE, 2001).

In order to do this, one of the key policies created to facilitate
this process in South Africa was Curriculum 2005 (C2005), which:

... envisaged for general education a move away from a racist, apartheid, rote
learning model of learning and teaching to a liberating, nation-building and
learner centred outcomes-based one. In line with training strategies, the re-
formulation is intended to allow greater mobility between different levels and
institutional sites, and the integration of knowledge and skills through "learning pathways" (DoE, 2001, p. 9).

In addition to this, C2005 also defined a set of critical and develop-
mental outcomes that are intended to overarch all programme
development. All learning programmes and assessment standards
in curriculum design are required to express these critical outcomes
in the various defined fields of learning, of which mathematics is one.
The critical outcomes include skills such as problem solving, critical
thinking, working in teams, communicating, and using science and
technology (DoE, 2001). The principles of OBE have been employed
in laying out these outcomes in the curriculum and underpin the
design and intended implementation of the new curriculum.

This implementation has been wrought with challenges, one
of which has been and continues to be the training of teachers.
Teachers are ultimately responsible for defining and delivering the
curriculum at classroom level (Hargreaves, 1989), and a grasp of the
relationship of teachers to the curriculum and to curriculum reform
is therefore vital. Teachers' beliefs and knowledge of a subject may
have a direct impact on their decisions, which in turn could affect
the classroom instruction they embark on (Fenema & Carpenter,

Research projects that have been carried out in South Africa since
the introduction of OBE and the new curriculum (see, for example,
Howie, 2002; Howie, Barnes, Cronje, Herman, Mapile & Hattingh,
2003; Barnes, 2004; Venter, Barnes, Howie & Janse van Vuuren, 2004)
indicate that the type of classroom instruction dominating in many
mathematics classrooms in South Africa does not resemble the intended
curriculum or philosophy as outlined in our reform policy documents.
From the background I explained in section 2, I am speculating that
this lack of intended change may relate to the level of subject matter
knowledge that our teachers exhibit. We know from existing literature
that a strong relationship between teachers' content knowledge and
how they teach has certainly been empirically established in research
done in the United States (see section 4), predominantly in elementary
and primary schools. In South Africa, the work being done by Jill Adler and her colleagues (see section 2) is focused on middle and senior school mathematics with in-service training. The gap I have therefore identified in the research is one that focuses on pre-service teachers in the secondary (high school) phase. I am therefore interested in investigating with such candidates the nature of a relationship between their subject matter knowledge and how this manifests in their classroom practice as pedagogical content knowledge.

Beyond a personal interest, I believe this research can have an impact on the way we train our pre-service mathematics teachers for the FET phase, as well as inform the continued support we could provide for them during their first few years of teaching. The debate about what mathematics content, the extent thereof and the balance between teaching mathematics content and methodology should be included in pre-service courses has been under way for some time now. My intention is that the study should produce rich data that will help us further understand the influence of subject matter knowledge on classroom practice and pedagogical content knowledge. This in turn will hopefully shed more light on furthering our progress in solving the quest for optimum pre-service training of mathematics teachers. Of course, if we can produce more effective mathematics teachers, our opportunities to improve learner achievement are much greater. From the results of the TIMSS studies conducted in 1995, 1999 and 2003 (see, for example, Howie, 2002), we also know that this is a domain within education which remains a great challenge to our education system. With the introduction of Mathematical Literacy as a compulsory subject for all FET learners from 2006, the need for effective mathematics teachers is even more foregrounded.

In summary, the rationale for the study lies embedded in a personal interest and experience of working with and training mathematics teachers, an empirical gap in the research literature on teachers in the secondary phase, and an intention to add value to the pre-service training programmes of secondary mathematics teachers at tertiary institutions. The research questions that will guide the inquiry are now discussed.

### 3.2 Research questions

The research questions configured to direct the study consist of a main research question that has been divided into subsidiary questions that will help to operationalise the inquiry. The main research question is as follows:

*How does the mathematics subject matter knowledge of pre-service secondary mathematics teachers manifest in their classroom practice?*
To address this main question, the following subsidiary questions will guide the inquiry:

a) What is the level of subject matter knowledge of mathematics that the teachers demonstrate?

b) How did teachers acquire this level of subject matter knowledge?

c) What is the relationship between their subject matter knowledge and their pedagogical content knowledge as displayed/demonstrated in their classroom practice?

4 Conceptual framework

Subject matter knowledge and classroom practice are the two main concepts being studied within this research project. These are first briefly defined, followed by a visual representation of how I envisage depicting each concept and interpreting the relationship between them in the study.

4.1 Subject matter knowledge (SMK)

Leinhardt and Smith (1985) offer a basic definition of subject matter knowledge (SMK) that is still quoted by more recent researchers in this domain (Hill, Schilling & Ball, 2004). They define it as including “concepts, algorithmic operations, the connections among different algorithmic procedures, the subset of the number system being drawn upon, the understanding of classes of student errors, and curriculum presentation” (p.247). Both Shulman (1986) and Grossman et al, 1990) expanded this definition to include the syntactic and substantive structures of a subject. Drawing on the work of Schwab (1978), they identified substantive structures as the different ways in which the fundamental principles and concepts of a discipline are organised (Shulman, 1986) that guide inquiry in the field and enable one to make sense of the data (Grossman, 1990). The syntactic structure relates to the set of rules that assist one in determining what is true or false, valid or invalid within a discipline (Shulman, 1986). New knowledge or claims can be deemed legitimate or unwarranted through these rules. Syntactic structures also consist of the tools of inquiry within a discipline (Grossman et al, 1990). Grossman et al (1990) then also included an additional dimension into their view of SMK that relates to teachers’ beliefs about and orientation towards the subject matter. In her work, Ball (1988b) makes a differentiation between knowledge of mathematics (knowledge of concepts and ideas, and how they work) and
knowledge about mathematics (e.g., how one decides that a solution is correct). Grossman et al (1990) refer to these two collectively as content knowledge for teaching.

For this purpose of this study, I found the four dimensions of SMK as presented by Grossman et al (1990) to be the most appropriate in defining this construct. These are: content knowledge for teaching, substantive knowledge, syntactic knowledge and beliefs of teachers. In my opinion, these dimensions comprehensively cover the views of SMK produced by researchers in this domain. For the construct of SMK, I have therefore developed the following definition:

Subject matter knowledge is the teachers’ knowledge of central facts, concepts, ideas and principles in mathematics, how they view these as being organised and relating to each other, and how they are able to make use of this knowledge in arriving at and evaluating correct claims, representations and solutions. It also encompasses teachers’ beliefs about the discipline of mathematics and their orientation towards the subject matter.

### 4.2 Classroom practice (CP)

This is, of course, a much broader term to define and in general terms could be understood to be what happens in a classroom. The European literature (see, for example, Goffree, Oliveira, Serrazina & Szendrei, 1999) often describes classroom practice by the components of the so-called “didactical triangle” (p. 156): the learner, the teacher and the subject matter, and the interaction among these components. This practice includes classroom management, administration, instructional practices, discipline, assessment practices, questioning techniques, communication between teachers and learners, time on task, planning, learning environment and media, to mention a few.

In their research on classroom mathematical practices, Cobb, Stephan, McClain and Gravemeijer (2001) use an interpretative framework consisting of both a social and a psychological perspective to analyse the communal and individual mathematical activity and learning.

<table>
<thead>
<tr>
<th>Social perspective</th>
<th>Psychological perspective</th>
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<tbody>
<tr>
<td>Classroom and social norms</td>
<td>Beliefs about own role, others’ roles, and the general nature of mathematical activity in school</td>
</tr>
<tr>
<td>Sociomathematical norms</td>
<td>Mathematical beliefs and values</td>
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<tr>
<td>Classroom mathematical practices</td>
<td>Mathematical interpretations and reasoning</td>
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</tbody>
</table>
Their social perspective draws on work from the sociocultural theory (e.g. Cole, 1996; Lave, 1988; Rogoff, 1997) and ethnomethodology and symbolic interactionism (Blumer, 1969) as they have been adapted for use in mathematics education (cf. Bauersfeld, Krummheuer & Voigt, 1988, cited in Cobb et al, 2001). Their psychological perspective has traces of constructivism (Piaget, 1970; Steffe & Kieren, 1994; Thompson, 1991) and accounts of intelligence (e.g. Hutchins, 1995; Pea, 1993). In their analytical framework, though, the psychological perspective relates to the learners in the classroom and their individual learning. I intend to use this framework in interpreting classroom practice within this inquiry, but to use the psychological perspective in relation to the pre-service teachers participating in the study and not the learners.

### 4.3 Visual representation of conceptual framework

How the student views and uses mathematical:

```
Subject matter knowledge (SMK) → Classroom practice (CP)
                             ↓
Social perspective          Psychological perspective
                             ↓
Concepts                    Facts, Principles, Ideas
                             ↓
How SMK manifests in CP
```
5 Preliminary literature review

This preliminary literature review informs the proposed theoretical framework for the study. An overview of developments relating to research within mathematical content knowledge is first outlined, followed by a synthesis of recent empirical studies that are relevant to this domain. The focus on primary and elementary school is evident in the available literature, supporting my intention to attend to the gap on the SMK of secondary pre-service teachers. A similar study has also not previously been conducted in a South African context. Within South Africa, with the exception of the work being led by Jill Adler on the Quantum project, there is not much empirical data available on the SMK of pre-service teachers. Most of the current literature being published stems from the United States and the United Kingdom. As these are both developed countries that have been in the process of educational reform for approximately ten years longer than we have, our developing context should present an interesting comparison to their results.

5.1 Brief historical overview

The second half of the 1980s proved to be an important watershed for research focusing on the content knowledge of teachers. In the 1960s and 1970s research carried out had mainly been in the form of quantitative studies (e.g. Begle, 1979; Eisenberg, 1977) as cited in Mewborn (2001). These studies endeavoured to demonstrate a connection between teachers’ knowledge (such as number of mathematics courses taken, mathematics majors, grade-point average) and learner achievement. The studies, however, failed to find a statistical correlation between these two measures. Schwab’s differentiation (1978) between substantive knowledge and syntactic knowledge (see section 2.3.1) in the work he did in science education in the 1960s and 1970s still features prominently in most research concerning SMK (e.g. Leinhardt & Smith, 1985; Shulman, 1986; Ball, 1988, 1990; Grossman et al, 1990). It was Shulman’s work in the 1980s, though, that spurred more researchers in teacher education to turn their interests toward content knowledge.

Shulman (1986) delved into archives in the United States containing reports of state superintendents of education from the late 1800s. There he was able to find copies of tests used for licensing teachers at the county level. After surveying tests from various

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3 Although the focus of this study is on subject matter knowledge, I use the phrase “content knowledge” purposively here to denote the more comprehensive domain of pedagogical content knowledge and SMK.
states for that period, he remarked that 90–95 percent of the test covered content either on the subject matter taught or the knowledge base assumed to be needed by teachers. Only 5–10 per cent considered the pedagogical practice of the teacher. Reflecting on the evaluation of teachers being done in the United States in the 1980s, he found that the emphasis had shifted to the assessment of teachers’ capacity to teach, and that the assessment of subject knowledge was virtually non-existent. He started asking questions again about the content knowledge of teachers and more specifically how teachers transform their SMK into a form that makes it accessible to the learners they teach. This resulted in his seminal work on pedagogical content knowledge and a renewed surge of research focusing on the content knowledge (relating to their subject and their knowledge of pedagogy) of teachers.

Around the same time that Shulman and his colleagues from Stanford were conducting their work on content knowledge, Leinhardt and Smith (1985) were doing similar studies in the domain of mathematics instruction. They studied the nature, level and use of SMK among a set of expert and novice teachers, specifically within the topic of fractions. Their theoretical framework was based within cognitive psychology where they viewed the content of the teacher’s lesson as the product of a cognitive system representing knowledge in declarative and procedural forms (Anderson, 1983). Declarative knowledge consists of the known facts within a domain. Procedural knowledge focuses on the algorithms and heuristics that operate on those facts. Their results indicated that more experienced and competent teachers exhibited a more refined hierarchical structure to their knowledge. They also commented on the disparity evident between the teachers’ ability to express an algorithm and their lack of understanding of the underlying mathematical concepts and relationships.

Since the 1990s the amount of research foregrounding content knowledge seems to have gained momentum. In an historical overview of mathematics teachers’ content knowledge, Mewborn (2001) identifies major research genres in literature on teacher knowledge in the United States as it chronologically progressed. Literature published since the 1990s suggests that descriptive, comparative and studies focusing on teachers’ practice have dominated during the last three decades.

The descriptive studies embodied the strengths and weaknesses within teachers’ knowledge of particular topics, such as multiplication and division (Simon, 1993; Simon & Blume, 1994), rational numbers (Post, Harel, Behr & Lesh, 1991; Tirosh, Fischbein, Graeber & Wilson, 1999), functions (Even, 1993) and geometry (Batro
& Nason, 1996). Results of these studies proposed that while elementary teachers in general (mostly, but not always) tend to know the necessary facts and algorithms in school mathematics, they lack sufficient conceptual understanding thereof. Their knowledge appears “compartmentalised and fragmented”, hence not easily transferable within domains (Mewborn, 2001, p. 29).

The comparative studies compared the knowledge of US teachers with teachers from other countries (Ma, 1999), elementary and secondary teachers (Fuller, 1997), and pre-service and in-service teachers (Ball, 1991; Ball & Wilson, 1990). These studies showed some variation among variables, but generally also endorsed the low level of conceptual understanding of mathematics teachers (Mewborn, 2001).

Further qualitative studies researching small numbers of teachers in their classroom practice (Eisenhart, Borko, Underhill, Brown, Jones & Agard, 1993; Heaton, 1992, 1995, 2000; Thompson & Thompson, 1994, 1996, cited in Mewborn, 2001) more adequately embraced the complexity of teachers’ knowledge. These studies exposed the multiplicities of the relationship between knowledge and teaching practice. While a number of elementary teachers with a low level of content knowledge were more prone to “show and tell” type teaching, a number of teachers with strong content knowledge made just as much use of this methodology. It is my intention that this study will further unravel some of these complexities in this research genre with secondary pre-service teachers. Similar studies and studies that can further inform my investigation in this regard are therefore now presented.

### 5.2 Other empirical studies

Dewey (1983) claimed that “every study of subject thus has two aspects: one for the scientist as a scientist; the other for the teacher as teacher” (p. 273). Teachers do not just teach, they teach a specific subject. Their knowledge therefore needs to extend beyond just the tacit knowledge of that subject to a more explicit knowledge (Ball, 1991) that enables them to make the subject accessible to their learners. As this thesis gained momentum, researchers in various subjects started examining the implications of this for teaching and learning within their particular subjects. Literature from these subjects (other than mathematics) and SMK in general are first discussed followed by recent work in the domain of mathematics.

**5.2.1 General SMK**

It seems fitting to start with work regarded in this domain as seminal. Schwab’s contribution (1978) of substantive and syntactic
knowledge distinctions, as well as Shulman’s work (1986) on pedagogical content knowledge (PCK) have already been noted. These contributions play an important role in other studies currently being undertaken and cited relating to content knowledge of teachers. Another reference widely cited is that of Grossman et al (1990). Their work re-examines the assumption that the SMK necessary for teaching can be acquired through courses taken in the relevant university departments. They refreshed the research agenda on SMK by providing five important reasons why the examination of SMK is important to teacher educators. These included:

- the diversity of students’ backgrounds in terms of the SMK they arrive with;
- the quality of undergraduate coursework;
- the relationship between academic disciplines of the subject as studied at university level and the content to be taught at school level;
- the difference between SMK per se and SMK for teaching; and
- the impact of students’ previous learning (in terms of at school and in undergraduate courses) on their teaching.

The four dimensions of SMK that Grossman et al (1990) presented in their analysis have already been mentioned in section 2.3.1; content knowledge, substantive knowledge, syntactic knowledge and beliefs about subject matter. These will be used in my derivation of a quantitative measure and qualitative description of the SMK of each of the participants in my study. Findings from Grossman et al (1990) relating to each of the four dimensions will also inform my analysis and interpretation of the data in answering my research questions. Overall Grossman et al (1990) concluded that when teachers demonstrated a deeper knowledge, this resulted in more emphasis on conceptual explanations in their teaching. Drawing on the work of Leinhardt and Smith (1985), they also agreed that teachers who displayed a better organisation of knowledge tended to be more effective in their teaching.

5.2.2 Mathematical SMK

In my opinion, Ball’s work (later assisted by other colleagues) has made a good theoretical contribution to literature on SMK in mathematics from the United States during the last two decades. In her initial work, Ball (1988a) challenged three existing myths on the preparation of prospective mathematics teachers by studying 19 teacher education students’ knowledge of mathematics
relating to the topic of division. She analysed their substantive knowledge along three qualitative dimensions, namely the value of truth in their knowledge, the legitimacy of their knowledge and the connectedness thereof. She firstly challenged the myth that “traditional school mathematics is simple” (p. 32) by showing that even students majoring in mathematics struggled when required to work below the surface of simple maths. While these students could perform procedures, they seemed to lack the warranted understanding of the content. They would, for example, know how to “invert and multiply” when required to do division by fractions, but not be able to provide any mathematical explanation for why this procedure is valid (see section 2.1, where I experienced a similar phenomenon with my students). The second assumption she contested was that “elementary and secondary school math classes can serve as subject matter preparation for teaching mathematics” (p. 33). She found that when teacher candidates tried to respond to tasks and questions drawing on what they had learnt in school, they typically exhibited loose fragments in their knowledge and understanding. Most of them did not display meaningful understanding. The third myth she opposed was that “majoring in mathematics ensures subject matter knowledge” (p. 33). Some of the students in her study were mathematics majors and had obviously done more maths than some of the other students. Although these students appeared to know more (in that they got more of the answers right), the additional studies did not seem to afford them any significant advantage in explaining and connecting underlying concepts, principles and meanings. This work of hers is important in my study in that a departure point of this investigation is one that stands on the falsehood of these very myths.

Ball (1988b; 1990) then went on to develop a framework for understanding what prospective mathematics teachers know and believe when they enter teacher education. She used interviews and structured tasks to explore the students’:

- knowledge of and about mathematics;
- ideas about the teaching and learning of mathematics; and
- feelings about themselves in relation to mathematics.

She then presented the thesis that “teachers’ subject matter knowledge interacts with their assumptions and explicit beliefs about teaching and learning, about students, and about context to shape the ways in which they teach mathematics to students” (Ball, 1991, p. 1). She developed this argument in three parts. Firstly, she analysed past investigations of the role of teachers’ SMK in teaching mathematics. Secondly, she unpacked the concept of SMK for
teaching mathematics and what is entailed in finding out what teachers know; and finally she presented three case analyses of teachers’ understanding of mathematics as displayed in their teaching of multiplication.

Her work has since gone on to focus on the subject matter preparation of teachers (Ball & Cohen, 1999; Ball & McDiarmid, 1990), intertwining pedagogy with knowledge (Ball, 2002; Ball & Bass, 2000) and how to go about measuring teachers’ mathematics knowledge specifically for teaching (Hill et al., 2004). In my study, I envision drawing on this framework she has developed in terms of teachers’ knowledge of teaching and knowledge for teaching when analysing and interpreting my qualitative data.

The other research I have found useful in the domain of SMK in mathematics is the work of Tim Rowland and his colleagues in the United Kingdom.  Although they are working with primary school teachers, their study has largely influenced the design of my conceptual framework and supported my rationale for the study. Their research provided statistical evidence that sound knowledge of mathematics topics is associated with more competent teaching of mathematics in the case of pre-service primary school teachers (e.g. Rowland, Martyn, Barber & Heal, 2001). Similarly, they were also able to relate weak SMK with less competent teaching of the subject. When a similar study was carried out in Ireland, though (using the same instruments), they were not able to establish any significant association between a quantitative measure of the SMK of pre-service primary teachers and their teaching performance (Corcoran, 2005).

One part of the research project mentioned above shares a similar methodological design to mine in that they videotaped and analysed mathematics lessons prepared and taught by PGCE primary school pre-service teachers, in order to identify ways in which pre-service teachers’ SMK, or lack of it, is evident in their teaching. The codes they identified through their analysis process (Huckstep, Rowland & Thwaites, 2005) will assist me in establishing codes for the inductive process of my video data analysis.

This project also developed a 16-item mathematics subject audit instrument that was administered to all their trainee teachers in the PGCE course to establish some quantitative measure of their

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4 Their project is known as SKIMA (SKM in mathematics), and is ongoing collaborative work among researchers at the universities of Cambridge, London, Durham and York.

5 This includes topics that extend beyond those found in the primary curriculum
SMK (Huckstep et al, 2005). This instrument was later used in a study with Irish primary school teachers (Corcoran, 2005) and also adapted for an initial exploration into the SMK of a convenience sample of primary school mathematics teachers in Hong Kong (Tsang & Rowland, 2005). If I am able to get access to this mathematics subject audit instrument, I would also like to include it as an additional quantitative measure for establishing the level of SMK of my case studies, as well as for potential comparative purposes with these other countries.

6 Research design

As already outlined, the purpose of this study is to investigate how the SMK of mathematics teachers manifests in their classroom practice. This naturally implies firstly a detailed understanding of their SMK and insight into their classroom practice. A nested mixed-methods approach has therefore been chosen to facilitate this process. Such an approach is one in which qualitative and quantitative strategies are used to collect and analyse data in a single study (Creswell, 2003). Both strategies may carry equal weight within a study or one method may be “nested” within another method (Tashakkori & Teddlie, 1998, cited in Creswell, 2003), as is the case in this design. Qualitative and quantitative data may be collected simultaneously (concurrently) or sequentially, in which case the researcher may further explore findings of one method with the use of another (Creswell, 2003). The mixed-methods approach therefore allows for the use of both predetermined and emerging methods, open- and closed-ended questions, and both statistical and text analysis (Creswell, 2003).

When this study was first conceptualised, I had intended to measure the SMK of the teachers using only a quantitative instrument. Subsequent readings in the literature led me away from this though and to the conclusion that the design of the study would benefit from a more dominant qualitative nature, with a smaller quantitative component nested in the design. Grossman et al (1990) report on how the earliest research on teacher SMK tried to identify statistical relationships between the knowledge of teachers and the achievement of their learners. The SMK of teachers was represented either as the number of classes that a teacher had taken in the subject, their grades obtained in the subject or their score on a standardised achievement/performance test. The majority of studies showed no significant relationship, though, and it was suggested that perhaps teacher SMK had not been adequately conceptualised (Byrne, 1983, cited in Grossman et al, 1990) and that it
is a complex phenomena that encompasses more than what can be measured on a test or by the level or grades of a teachers’ qualification. As the SMK of teachers is so intertwined with their immediate knowledge of what is happening in the classroom (Calderhead & Miller, 1985), it is also difficult in my opinion to study the SMK of teachers outside of the decisions they take within the classroom. To therefore operationalise my main research question, I have chosen a predominantly qualitative design with for this study with an interpretive epistemology as outlined in the following section.

6.1 **Paradigm**

As this study seeks to understand the relationship between constructs central to its participants and their experiences, it is situated within an interpretive paradigm. This type of paradigm views the world as an emergent social process (Burrell & Morgan, 1979) and aims to characterise how people experience the world, ways in which they interact together and the settings in which these interactions take place (Packer, 2007). It seeks to explain behaviour from the individual’s point and understand the subjectively created world “as it is” (Burrell & Morgan, 1979). In this inquiry, the setting is the classroom where interactions between the teacher and the learners will be studied in order to try to understand how the SMK of the teacher manifests itself in their classroom practice.

Methodologically, the positioning of my research within this paradigm assumes a participatory stance for myself as the researcher and requires the description of specific cases through narrative articulation and interpretation. In terms of epistemology, an underlying assumption I bring into the inquiry is that people have and use interpretive schemes that must be understood. This therefore highlights the need to explicitly articulate the character of the local context within the study. Ontologically, the interpretive paradigm locates the participants in this study, as well as the constructs being investigated, within inter-subjective social fields (in this case the educational context) that structure and constrain activity (Packer, 2007).

As the researcher in the study, while I do not intend taking a participatory position as one of the actual participants, I can also not claim to be an observer. I am the lecturer of the participants and this therefore entails me working closely with them throughout the duration of the year. It is my intention in the development of the dissertation to depict as clearly as possible my relationship with the participants in the dual roles of researcher and lecturer.
6.2 Research approach

To provide depth in answering the research questions, I have selected a convenience sample of four participants. The participants have been chosen through convenience sampling based on their willingness to be part of the study, as well as the fact that they are all (there are only four in the class) enrolled for the Mathematics Specialisation course in obtaining a Post Graduate Certificate in Education (PGCE) at the University of Pretoria. They complete this one-year post-graduate qualification on completion of their undergraduate degrees, in order to qualify as teachers. I am the mathematics specialisation lecturer for all of the participants. The backgrounds the participants start the PGCE with differ, but they all followed a similar route to qualify in becoming teachers.

As has already been mentioned, the relationship between their raw SMK that they enter the PGCE with and their initial classroom practice is the focus, rather than a developmental aspect of these constructs over time. The data collection period will include their initial preparation at the university as well as their school-based periods of practice teaching.

This setting simulates that of a case study as defined in the literature on research designs (e.g. Adelman, Jenkins & Kemmis, 1980; Guba & Lincoln, 1981; Merriam, 1988; Cohen, Manion & Morrison, 2000). Merriam (1988) cites definitions from various authors who support this, such as a case study being defined as “the examination of an instance in action” (MacDonald & Walker, 1977, p. 181) and a process “which tries to describe and analyse some entity in qualitative, complex and comprehensive terms not infrequently as it unfolds over a period of time” (Wilson, 1979, p. 448). The context of this inquiry is also dynamic and provides a unique example of real teachers in a real classroom situation (Cohen et al, 2000).

More specifically, Bennet and George (1997) refer to the type of case study research I plan to use as the “method of structured, focused comparison” (p. 2). They make the following point:

Comparative case studies can use within-case analysis of individual cases as well as case comparisons to assess and refine existing theories, and more generally, to develop empirical theory. The method of doing is “structured” in that the same general questions are asked of each case in order to guide the data collection, thereby making possible systematic comparison and cumulation of the findings of the cases. The method is “focused” in that it deals with only certain aspects of the cases; that is, a selective theoretical focus guides the analysis of the cases.

The theoretical focus that guides these case studies is to establish the existence, nature and extent of any relationship between the
SMK and classroom practice of the participants. The point of departure will be to first examine that relationship within the individual cases before comparing the SMK and the classroom practice across the different cases. This will be done with a view to contributing to the body of knowledge that already exists within this domain of mathematics education. Bennet and George (1997) identify this type of theory-building objective as having “heuristic purposes” (p. 5). This includes searching for new variables, hypotheses and causal mechanisms and paths, through an inductive process. They propose that the structure and focus of such studies are more easily attained when a single investigator plans and carries out all of the case studies. The data collection and analysis are further outlined in the sections that follow.

6.3 Data collection strategies

This study has been placed within an interpretive paradigm, thereby drawing on mainly qualitative data collection and analysis methods. However, a quantitative measure of the pre-service teachers’ SMK will be included, as mentioned. This will be obtained from the mathematics content analysis test they write on entering the Mathematics Specialisation of the PGCE course. The primary source of data will come from video-taped presentations and lessons by the pre-service teachers during their university-based learning period and their school based period respectively. Interviews with the teachers will also be an invaluable source of rich data, and document analysis will be used to provide more details on each case for the purpose of narrative descriptions in the final report. They will also support the process of triangulation in ensuring the trustworthiness of the data. A more detailed description of each data source is provided below.

6.3.1 Video data (classroom observations and class presentations)

This data will be collected over a period of approximately six months. The pre-service teachers will first be video taped conducting two micro-lesson presentations during their first few specialisation lectures at the university. Thereafter, approximately three lessons during their first school-based period (of eight weeks) will be taped and analysed. For these three lessons, one will be taped during the first two weeks of their school-based period, another during the middle two weeks and the last one during the final two weeks. Finally, one of their mid-year assessments (which involves
presenting a lesson at the university to their peers in their specific subject field) will also be taped. This will provide data on the strategies they employ during their teaching of mathematics, as well as insight into their SMK and pedagogical content knowledge. The data from these videos will then also be used in the interviews (discussed below) in order to gain further insight into the planning, thinking and decisions that teachers take during their teaching. The intention here is to gain some understanding about how their subject matter influences and plays out in their classroom practice.

6.3.2 Interviews

I intend conducting interviews with each of the participants. It is envisioned that these interviews will be done on a one-to-one basis (Creswell, 2003) and all audio taped with the permission of the students.

This form of data collection will be used for two main purposes: firstly with a view to enabling the participants to discuss their interpretations of video data presented to them of their lessons, and secondly to allow them to express their opinions, experience, attitude towards and perceptions of mathematics (Cohen et al, 2000; Gay & Airasian, 2003). This will also enable me to gain insight into their SMK on the topic taught in the video-taped lessons and how they gained this level of knowledge.

With these aims in mind, I intend constructing an interview protocol in the form of what Patton (1980) refers to as the “interview guide approach” (cited in Cohen et al, 2000) and what Krathwohl (1993) calls “partially structured” interviews. In this type of interview, the area of discussion is chosen and the questions are formulated in advance, but the interviewer decides on the sequence of the questions during the course of the interview. The questions are mainly open ended and the interviewer also has the liberty to add questions or to modify them as she sees fit, depending on the responses of the participants. The types of questions I would like to include contain a mixture of both direct and indirect forms and were mostly what Kvale (1996 as cited in Cohen et al, 2000) terms “process questions”. These are questions that either directly or indirectly ask for information that follow-up on a topic or ideas, that probe for further information or responses, and that ask respondents to specify and provide examples.

6.3.3 Document analysis

Documents that will be analysed during the course of the study include a mathematics content analysis test and lesson preparation
done by the pre-service teachers. The mathematics content analysis test is written by all the prospective PGCE Mathematics Specialisation students at the start of the course. This gives me as the lecturer an opportunity to get a basic understanding of their mathematical literacy and for both the pre-service teachers and myself to identify content areas that may require extra attention during the course of the year. This content analysis will give some quantitative measure and comparison of the mathematical literacy levels of the different participants.

The lesson preparation documents will be the planning that teachers do prior to the lessons. It is hoped that these will provide meaningful insight into the preparation that teachers put into their lessons and how this either corresponds with or contradicts what actually plays out during the course of the lessons. These documents will also offer further insight into the SMK of the participants.

6.4 Data analysis

Since the inception of this study, I have found my ideas on data analysis going off on varying tangents as I have worked through more literature on other empirical studies conducted in this domain. I think that this will continue to happen as my reading base continues to expand. The dynamic nature of interpretive, qualitative studies allows and encourages this, in my opinion. The suggestions I provide in this proposal though are therefore merely a guideline currently for me and somewhat general and superficial. However, this will grow and develop as the study progresses.

The quantitative analysis of the mathematics content test will be done using descriptive statistics and individual item analysis. This will be facilitated by the SPSS software package. I intend analysing the qualitative data (video data and interviews) using an inductive but, to a lesser extent, also a deductive approach. The deductive approach will facilitate the indicators and categories already identified in the literature (see, for example, the dimensions of SMK stated in section 4). The inductive approach is to allow for the formulation of new themes that may come out of the data. This means that the scheme for analysing the themes associated with the content become apparent during the analysis itself and are not predetermined as is the case with the deductive approach. This type of inductive analysis (Miles & Huberman, 1994; Creswell, 2003; Gay & Airasian, 2003) allows me to construct patterns that emerge from the data in order to make sense of them. In such an analysis, one usually starts with a large set of issues and, through an iterative process, progressively narrows them down into small
important groups of key data. From this data, variables are then identified through further examination and analysis that can be interpreted and discussed. This therefore creates a multistage process of organising, categorising, synthesising, interpreting and reporting on the available data (Gay & Airasian, 2003).

To encompass both approaches, and to efficiently manage that analysis, I have chosen two software packages. The video data analysis will be handled by a tool known as Transana. This software permits me to sort my data into series (each case will be a separate series) that each contains a set of episodes (each lesson will constitute a separate episode). The episodes are then analysed and coded by me, with the unit of analysis being actual video clips or segments of the video deemed to be interesting or relevant to the study. These are coded and stored in collections, which later develop into emerging themes for the study. For the interviews, I will use Atlas-TI, which enables me to code, organise and sort transcribed data in a similar process to the one outlined above.

6.5 Issues of validity and reliability

As this study entails both quantitative and qualitative data, the terms used to depict validity and reliability are not limited to these two terms, which are more commonly associated with quantitative data. When working with qualitative data, the terms trustworthiness, dependability, transferability and credibility are also used. To try and simplify the process of taking cognisance of all of these, this section has been divided into three sub-sections, which deal with triangulation, and ten the qualitative and quantitative data. Issues relating to validity and reliability in relation to each of these sub-sections are outlined therein.

6.5.1 Triangulation

Cohen et al (2001) define triangulation as the “use of two or more methods of data collection in the study of some aspect of human behaviour” (p. 112). For Denzin (1994), the concept of triangulation involved the combination of data sources to study the same social phenomenon.

Denzin (1994) distinguished between four basic types of triangulation, two of which will be applied in this study. These include data triangulation and methodological triangulation. *Data triangulation* involves the use of a variety of data sources in a study and in this particular case; these are present in the form of video data (observations), interviews and document analysis. *In methodological triangulation*, both quantitative and qualitative methods are used
to study a research problem. The inclusion of these methods has already been presented in the sections above. Each approach is now individually considered.

### 6.5.2 Qualitative data

The qualitative data being collected is in the form of video data and interviews. Both will be electronically recorded and transcribed. Participants will have the opportunity to review these transcriptions at the end of the entire data collection period to ensure accuracy and provide additional research data. My own reflections, thoughts, observations and uncertainties during the course of the study will be recorded in a journal to provide an audit trail and assist me in identifying and acknowledging possible personal biases and preferences that will affect the data analyses (Gay & Airasian, 2003). A further source to increase the internal validity of the qualitative data will be to draw on literature discussing (where possible) similar and conflicting findings to the outcome of my study (Eisenhardt, 2002). It is also envisioned that the theory-building process (Bennet & George, 1997; Eisenhardt, 2002) will be strengthened in its validity and generalisability by the high number of case studies (seven in total) being depicted and compared.

### 6.5.3 Quantitative instrument

Validity procedures set in place for the mathematics content analysis test include content- and criterion-related validity. Content validity requires that the instrument must show that it fairly and comprehensively covers the domain or items that it purports to cover (Cohen et al, 2000). Gay and Airasian (2003) state that content validity is achieved when the two components, namely *item validity* and *sampling validity*, contained therein are both met. *Item validity* deals with whether the test items are relevant to the measurement of the intended content area, while *sampling validity* is concerned with how well the test samples the total content area that is being tested. This is mainly determined by expert judgement where an “expert” in the field reviews the process of compiling the test as well as the content of the test (Gay & Airasian, 2003). The test has already been set up and validated by two experts in the field of mathematics education.

Criterion-related validity (Cohen et al, 2000; Gay & Airasian, 2003) endeavours to relate the results of one particular instrument to a second measure. In this case, the data obtained from the interviews and the video-taped data, as well as the lesson preparation docu-
ments can be used as a further source of comparison for criterion evaluated in the content analysis test.

Reliability is a synonym for consistency and replicability over time, over instruments and over groups of respondents, and is therefore concerned with precision and accuracy (Cohen et al., 2000). The two types of reliability that can be measured in relation to the content analysis instrument are the stability of the instrument and the internal consistency of it. This will be verified once the data from the test is finalised. Statistics available from previous students indicates that the test holds a sufficient degree of reliability.

### 6.6 Ethical considerations

As I am the lecturer of the participants, I need to ensure that they do not feel coerced or compelled to be part of the study. I therefore explained the nature of the study to them and indicated that their willingness to be part of the study would in no way impact on their participation in the Mathematics Specialisation module. In approaching the participants, the following steps were followed (Gay & Airasian, 2003):

- The purpose and an outline of the study were provided to them and they were asked if they would consider availing themselves as participants.
- It was emphasised that their participation is entirely voluntary.
- They were promised full confidentiality and anonymity on events that take place during the study, but were asked to give full release on the video data for use in public domains such as training and presentations.

I have not, and am not intending to obtain ethical clearance from any of the learners who will be present in the classes I am video taping. The reason for this is that they are not the focus of the study. Any data used as evidence within or in presentations arising from the study will be suitably “doctored” or edited to ensure the anonymity of the learners. It is envisaged that this will mainly be done through an editing technique known as blurring.

### 7 Limitations of the study

I am confronted at this stage with two main limitations of this study. Firstly, the fact that I am both a lecturer of the participants and the researcher could have an impact on the investigation. Currently, I am still in the process of further engaging with available literature
relating to this issue. The advantage of this situation is that it affords me even further insight into the participants outside of the data being collected. I envision this contributing to the overall depth and richness of the case studies.

The second limitation pertains to the lack of generalisability of case studies. While this study is restricted to one tertiary institution, the participants have gained their undergraduate degrees at a variety of different institutions and represent a variance of gender, race, cultures and languages. It is also not my intention to generalise these results of individual cases but to add to the body of knowledge on the relationship between SMK and classroom practice.

References


Howie, S.J. (2002). ‘English language proficiency and contextual factors influencing mathematics achievement of secondary school pupils


